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Exponentiated Non-Negative Poisson Distribution: Application & Model

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Abstract: The Exponentiated non-negative Rayleigh Poisson distribution is introduced in this research work by combining A new generalization of Rayleigh distribution; characteristics and applications with The Exponentiated G Poisson model. This study's primary goal is to use the power transformation technique to increase the Exponentiated G. Poisson distribution's flexibility. The new proposed model's probability density function, survival function, and hazard function are shown graphically. We examine this new distribution's characteristics, paying particular attention to its moments, skewness, kurtosis, mode, and quantile function. The probability-weighted moments, order statistics, R'enyi, entropies, and the residual life function have all been covered. We computed the log-likelihood, Bayesian, Akaike, and corrected Akaike information criteria and Hennan-Quinn information criteria of the following distributions: the newly proposed Exponentiated Rayleigh Poisson distribution, the Compound Rayleigh distribution, the Exponentiated Chen distribution, the Exponentiated Exponential distribution, the Exponentiated Inverted Weibull distribution, and the Generalized Rayleigh distribution. We discovered that the newly proposed model has smaller values than the others. For model validation, we have examined the TTT plot, Q-Q plot, Kolmogorov Smirnov test, and P-P plot of the suggested distribution. We contrasted the proposed model's estimated distributed function CDF and empirical distribution CDF with those of five other models. For illustrative purposes, an actual dataset is examined. By using various methods and resources, the new family's significance and adaptability are demonstrated. The paper is concluded with a final conclusion.

Index Terms - New Generalization of Rayleigh Distribution, Exponentiated G Poisson Model, Probability Weighted Moments (PWM), Order Statistics, Maximum Likely-hood Estimation (MLE)

I. INTRODUCTION

Numerous generalized distributions based on various modification techniques have been developed in recent decades. These alteration techniques call for the inclusion of one or more parameters to the base model that might improve the modeling of real-world data's adaptability. Even though the analytical solutions are highly complex, many of these techniques are now accessible because to modern computing technology [8]. Lifetime data modeling and analysis are essential in many practical sciences, including engineering, finance, and medicine. To represent such data, a variety of lifetime distributions have been employed. The assumed probability model or distributions have a significant impact on the analysis's quality. As a result, significant work has gone into creating extensive classes of standard probability distributions as well as pertinent, statistical techniques. Nonetheless, there are still a lot of significant issues where actual data deviates from any of the conventional or classical probability models [20]. In order to increase the flexibility of modeling data from a practical perspective, a new family of distributions has been presented over time to generalize different distributions by compounding well-known distributions. [12] presented a broad class of distributions derived from the beta random variable's logit. The "transformer" is a random variable X that is used to transform another random variable T, the "transformed," in order to generate families of continuous distributions [6].investigated a novel three-parameter distribution driven mostly by life concerns [25]. [18] suggested the exponentiated Kumaraswamy distribution, a generalization of the Kumaraswamy distribution. A new general family of distributions derived from the logit of the gamma random variable, a specific case of the gamma uniform distribution, is examined in [28], whereas [24] presented a novel distribution produced by gamma random variables. [21] extended the transmuted G family by proposing a new family of continuous distributions known as the exponentiated transmuted G family. [16] discovered a two-parameter family of distributions on (0,1) that shares many characteristics with beta distributions and offers several tractability benefits. The Marshall-Olkin Kumaraswamy distribution is a new family of continuous distributions that was established in [26], and a new class of continuous distributions with two additional form parameters known as the generalized odd loglogistic distribution family.

There are other ways to develop a new probability model, but combining a legitimate probability distribution with a family of probability distributions is the most popular approach. The "Exponentiated G Poisson (EGP) Model" and "A new generalization of Rayleigh distribution: properties and applications" were combined to create the "Exponentiated Rayleigh Poisson distribution (ERP)" that we suggested in this work. This study's primary goal is to use the power transformation technique to increase the Exponentiated G. Poisson distribution's flexibility. Adding an additional parameter to the parent model typically increases flexibility and enhances goodness of fit. Section 2 provides a clear presentation of the CDF and PDF functions, Reliability/Survival and Hazard Rate Functions (HRF). The quantile and median functions, mode, skewness, Kurtosis, moments, moment-generating function, probability-weighted moments, residual life function, order statistics, and entropy are among the significant statistical characteristics that are examined in Section 3. The Maximum Likelihood Estimate (MLE) method is used in section 4 to estimate the model parameters. In Section 5, actual data analysis are examined to validate theoretical conclusions in various contexts. Section 6 provides some concluding observations.

II. Exponentiated Non-negative Rayleigh Poisson Distribution

The Truncated Poisson Distribution and the Exponentiated G family are compounded to produce the Exponentiated G Poisson family. The suggested family of distributions extends a number of standard distributions [15], with the probability and cumulative distribution functions provided as

$$F(x) = 1-e^{-\beta[G(x,\theta)]^{\alpha}} / 1-e^{-\beta} \quad ; x>0, \ \alpha > 0, \beta > 0, \theta > 0 \tag{2.1}$$

where β>0, θ>0, and α>0 are the shape parameters.

$$F(x) = \alpha\beta g(x)[G(x,\theta)]^{\alpha-1} e^{-\beta[G(x,\theta)]^{\alpha}} / 1-e^{-\beta} \qquad x>0, \ \alpha > 0, \beta > 0, \theta > 0 \tag{2.2}$$

The Rayleigh distribution is one of the most popular among probability distributions. Introduced in 1880, it was said to be a unique instance of the Weibull distribution. It is essential for modeling and analyzing life-time data, including survival, project loading, and reliability analysis communication theory [4].

$$g(x) = 2\,\theta^2 x e^{-(\theta x)^2} \; ; x>0 \, , \theta > 0 \tag{2.3}$$

Exponentiated Poisson The probability density function is based on the Rayleigh distribution

$$f(x) = {}^{2\alpha\beta\theta^2 x} e^{-(\theta x)^2} \, [e^{-(\theta x)^2}] e \, -\beta[1-e^{-(\theta x)^2}] / 1-e^{-\beta} \tag{2.4}$$

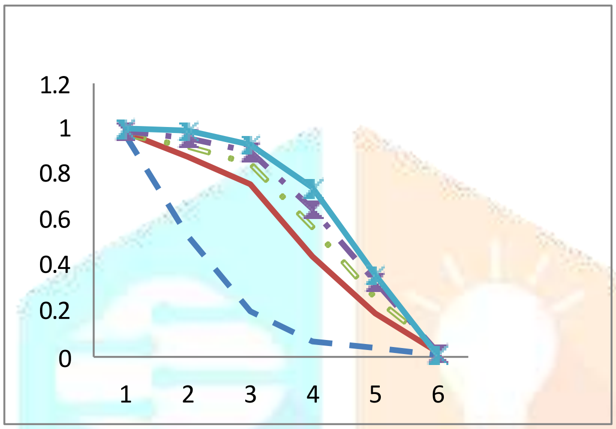
2.1 Survival Function

The survival function is the likelihood that there will be no failure before time t. The proposed model's survival function is R(x) = 1 - F(x).

$$= 1- \frac{1-e^{-\beta[G(x,\theta)]^{\alpha}}}{1-e^{-\beta}} ; x>0,\ \alpha > 0, \beta > 0, \theta > 0 \qquad (2.5)$$

As results in a more favorably skewed distribution, when α. This shows that the ERP distribution's PDF is well-suited for non-normal and skewed data. Increasing the value of α and β.

Figure 1 (left panel) shows graphs of density at θ = 10 for different values of the density $\alpha < 1$ and $\beta < 1$ is reversed J-shaped.



2.2. Hazard Function

The risk function represents the instant rate of failure at a specific time t. Thus, the proposed model's hazard function is:

$$H(x)= \frac{f(x)}{R(x)} = 2\alpha\beta\theta^2 x e^{-(\theta x)2}[e^{-(\theta x)2}]e^{-\beta[1-e^{-(\theta x)2}]} / 1\text{-}e^{-\beta[G(x,\theta)]\alpha} \qquad (2.6)$$

ERP distribution's hazard function, which is concave and convex.

III. Statistical Properties

This section has derived the major features of ERP distribution.

3.1. Useful Expansions

The distribution is generated from the generalized binomial series. For |Z| < 1, *n* > 0 we have

$$(1-z)^i = \sum_{m=0}^{\infty} (-1)^m \binom{i}{m} Z^m \qquad (2.7)$$

A power series extension of an exponential function is

$$e^{-az} = \sum\sum_{n=0}^{\infty} \frac{(az)^n(-1)^n}{\text{n!}} = 0 \qquad (2.8)$$

Equation (4) uses the binomial theorems (7) and (8) to calculate the PDF of the proposed model.

$$f(x) = \sum_{m=0}^{\infty} \varepsilon_m \, x e^{-(1+m)(\theta x)2} \qquad (2.9)$$

Where,

$$\varepsilon_m = \sum_{n=0}^{\infty} \frac{(-1)^{m+n} 2\alpha\theta^2 \beta^{n+1}}{(1-e^{-\beta})n!} \binom{\alpha(n+1-1)}{m}$$

3.2. Quantile Function

Quantile functions are utilized in theoretical probability theory. It is a substitute for PDF and CDF that is used to calculate statistical metrics such as median, skewness, and kurtosis. It can also be used to create random variables. The quantile function is calculated as Q(u) = F^{-1}(u). The quantile function for the purposed model is given as:

$$Q(u)=\left[-\frac{1}{\theta^2}\ ln\left[1-\left\{\frac{1}{\beta}\ ln\left[1\ -\ u\left(1-e^{-\beta}\right)\right]\right\}^{\frac{1}{\alpha}}\right]\right]^{\frac{1}{2}} \qquad (2.10)$$

Where u is uniformly distributed. Sustaining u=0.5 in equation (10) yields the median

3.3 Mode

The mode represents the PDF's maximum repeated value. To compute the mode's necessary

and sufficient condition is;

$$\frac{df(x)}{d(x)} = 0 \text{ and } \frac{d^2f(x)}{d(x)^2} > 0$$

After applying the relevant conditions, we get

$$\frac{1}{x} - 2\theta^2 x\left[1 - e^{-(\theta x)^2}\left\{\frac{1}{1-e^{-(\theta x)^2}}\right\}\{(\alpha\ -\ 1)\ +\ \alpha\beta(1\ -\ e^{-(\theta x)^2})\}\}\right]=0 \qquad (2.11)$$

Equation (2.11) is nonlinear, and there is no analytical solution. It can be calculated quantitatively using the Newton-Raphson formula.

3.4. Skewness and Kurtosis

The skewness and kurtosis are statistical terms that characterize the distribution's features. Bowley's skewness [5] takes the form.

$$S_k = \frac{Q\left(\frac{3}{4}\right)-2Q\left(\frac{1}{2}\right)+Q\left(\frac{1}{4}\right)}{Q\left(\frac{3}{4}\right)-Q\left(\frac{1}{4}\right)}$$

Moors' kurtosis [5] is based on Octiles and could be written as

$$K_u = \frac{Q\left(\frac{7}{4}\right)-2Q\left(\frac{5}{2}\right)+Q\left(\frac{1}{4}\right)}{Q\left(\frac{6}{4}\right)-Q\left(\frac{1}{4}\right)}$$

Where Q (.) represents the quantile function specified in equation (11). To compute the statistical measure of the suggested distribution, 100 random samples are generated from the equation (12) at the beginning value $\alpha = 5.0$, β =2.0 and $\lambda = 10$. decreases. Different skewness and kurtosis values exhibit non-symmetrical and non-normal (non-mesokurtic) features.λ increase, whereas β and αWe analyzed the mean, median, mode, standard deviation, skewness, and kurtosis of random samples to determine the intended model's features. The model's value increases with the initial parameter values. The proposed model's standard deviation decreases as the values of the suggested model exhibits a unimodal skewed and non-normal distribution.

Parameters			Mean	Median	Mode	SD	Skewness	Kurtosis
α	β	θ						
1.0	1.0	6.0	4.8720	5.2580	3.8742	1.1281	-1.4963	5.1703
1.5	2.0	5.5	5.7392	6.1684	4.3158	1.1650	-2.2793	8.0997
2.0	3.0	5.0	5.5871	5.8931	5.7633	1.2693	-1.5387	4.9725
2.5	4.0	4.5	4.0443	4.0385	6.8542	1.8931	-0.1663	1.9906
3.0	5.0	4.0	2.3896	2.6297	5.0951	2.0541	0.8598	2.3419
3.5	6.0	3.5	3.1269	0.6889	6.9875	0.8763	2.0765	7.4357
4.0	7.0	3.0	0.3921	0.1241	5.7103	0.6541	3.9986	19.6778

3.5. Moments

Moments are mathematical functions that describe the characteristics of a distribution. Since $X \sim ERP(\alpha, \beta, \theta)$ the r^{th} The raw moment is defined as (using the value of f(x) from the equation (2.9)

$\mu_r{}' = E(X^r)$

$$= \sum_{m=0}^{\infty} \varepsilon_m \int_0^{\infty} x^{r+1} e^{-(1+m)(\theta x)^2} dx \quad (2.12)$$

After integrating (2.12), the suggested model's r^{th} raw moment is;

$$\mu_r{}' = \sum_{m=0}^{\infty} \frac{1}{2\theta^{r+2}(1+m)^{\frac{r+1}{2}}} \varepsilon_m (\frac{r}{2}+1) \quad (2.13)$$

Table 1 shows the mean and standard deviation of the suggested model with different parameter values. The lower incomplete moments, say $\tau_s(t)$, is given by;

$$\tau_s(t) = \int_0^r x^2 \ f(x)\ d(x) \quad (2.14)$$

Using the relationship (2.9) in equation (2.13), and applying the lower incomplete gamma function

$$\tau_s(t) = \sum_{m=0}^{\infty} \frac{1}{2[(1+m)\theta^2]^{(\frac{s}{2}+1)}} \varepsilon_m \gamma(\frac{s}{2}+1)(1+m)t^2\theta^2$$

Similarly, the conditional moment is defined as

$$\emptyset_s(t) = \int_0^r x^2 f(x)\, dx \quad (2.15)$$

Using the connection (2.9) in equation (2.15), then applying the upper incomplete gamma function and integrating equation (15), we get the value of the conditional moment

$$\tau_s(t) = \sum_{m=0}^{\infty} \frac{1}{2[(1+m)\theta^2]^{(\frac{s}{2}+1)}} \varepsilon_m \Gamma(\frac{s}{2}+1)(1+m)t^2\theta^2$$

$$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} E(X^r) \quad (2.16)$$

The MGF is calculated by using the result of equation (2.13) in equation (2.16)

$$M_x(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \sum_{m=0}^{\infty} \frac{1}{2\theta^{r+2}(1+m)^{\frac{r+1}{2}}} \varepsilon_m (\frac{r}{2}+1)$$

The Probability Weighted Moments (PWM) The Probability Weighted Moments were calculated using the following relation.

$$\emptyset_{r,s} = \int_{\infty}^{-\infty} x^r f(x) F(x)^s dx \quad (2.17)$$

Now, we get applied expansion of

$$|F(X)|^s = \sum_{k=0}^{\infty} \eta_k e^{-k(\theta x)^2} \quad (2.18)$$

Where, $\eta_k = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{(-1)^{m+n+k}(\beta m)^n}{(1-e^{-\beta})n!^s}$

Now using the relation (2.9) and (2.18) in equation (2.17) it becomes

$$\emptyset_{r,s} = \sum_{m=0}^{\infty}\sum_{k=0}^{\infty}\varepsilon_m\, \eta_k \int_0^{\infty} x^{r+1}\, e^{-(1+m+k)(\theta^2)} f(x)\, dx \tag{2.19}$$

3.6. Order Statistics

Let X(1) < X(2) < … <X(n) the random sample of ordered statistics of size n from the following ERP distribution with α, β and θ. The PDF of I[th] order statistic [David (1981)], has defined as

$$f(X_{(i)}(X_{(i)})) = \frac{f(x_{(i)})}{B(i,j-i+1)}\sum_{u=0}^{i-j}(-1)^u\binom{i-j}{u}F(X_{(i)})^{u+i-1} \tag{2.20}$$

substituting (2.9) and (2.19) in equation (2.20) where S is replaced by $\vartheta + i$-1. Then equation (2.20) becomes

$$f(X_{(i)}(X_{(i)})) = \frac{1}{B(i,j-i+1)}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{u=0}^{i-j}(-1)^u\frac{(-1)^{m+n+k}(\beta m)^n}{(1-e^{-\beta})n!^s}$$

$$= \frac{1}{B(i,j-i+1)}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{u=0}^{i-j}(-1)^u\,\eta_{(X_{(i)})}\frac{(-1)^{m+n+k}(\beta m)^n}{(1-e^{-\beta})n!^s}$$

$$= \frac{1}{B(i,j-i+1)}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{u=0}^{i-j}(-1)^u\,\eta^*_{(X_{(i)})}e^{-(m+k+1)\theta^2 x_i^2} \tag{2.21}$$

Where, $\eta^* = (-1)^u\binom{n-m}{u}\varepsilon_m\eta_k$ then, the moment of order statistics is

$$= \frac{1}{B(i,j-i+1)}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\sum_{u=0}^{i-j}\eta^*\frac{\Gamma\left(\frac{r}{2}+1\right)}{[(1+i+k)\theta^2]^{\left(\frac{r}{2}+1\right)}} \tag{2.22}$$

3.7. Maximum Likelihood Estimation

Estimates of maximal likelihood (MLEs) of the distribution's unknown parameters on x= (x1,….,xn) The observed sample value and the set of parameters $\ell(\alpha, \beta, \theta \mid x)$ the loglikelihood function of the parameter $\ell(\alpha, \beta, \theta)$ is given by

$$l = n\ln(2\alpha\beta\theta^2) - n\ln(1-e^{-\beta}) + \sum_{m=1}^{i}\ln(X_{(m)}) - \theta^2\sum_{m=1}^{i}x_m^2 \tag{2.23}$$

The maximum likelihood estimators of the parameters have been derived Differentiating with regard to parameters and equating to zero yielded maximum likelihood estimators of the parameters. Let

$$\frac{\partial l}{\partial \alpha} = \frac{n}{\alpha} + \sum_{m=1}^{i}\ln(\emptyset^m) - \beta\sum_{m=1}^{i}\emptyset^m\ln(\emptyset^m) \tag{2.24}$$

$$\frac{\partial l}{\partial \beta} = \frac{n}{\beta} - \frac{n}{e^{(\beta-1)}}\sum_{m=1}^{i}\emptyset^m \tag{2.25}$$

$$\frac{\partial l}{\partial \theta} = \frac{2n}{\theta} - 2\theta\sum_{m=1}^{i}x_m^2 + 2\theta(\alpha-1)\sum_{m=1}^{i}\omega_m x_m^2 - 2\alpha\beta\sum_{m=1}^{i}\omega_m\emptyset_m^2 \tag{2.26}$$

The maximum likelihood estimators of the parameters have been derived α, β and θ by Non-linear equations (2.24), (2.25) and (2.26), cannot be solved analytically to estimate unknown parameters. As a result, we directly use Newton-Raphson's iterative technique to the log likelihood function in equation (2.23) to estimate the value of unknown parameters.

$$O(\hat{\delta}) = \begin{matrix} \frac{\partial^2 l}{\partial \alpha^2} & \frac{\partial^2 l}{\partial \alpha \partial \beta} & \frac{\partial^2 l}{\partial \alpha \partial \theta} \\ \frac{\partial^2 l}{\partial \alpha \partial \beta} & \frac{\partial^2 l}{\partial \beta^2} & \frac{\partial^2 l}{\partial \alpha \partial \theta} \\ \frac{\partial^2 l}{\partial \alpha \partial \theta} & \frac{\partial^2 l}{\partial \beta \partial \theta} & \frac{\partial^2 l}{\partial \theta^2} \end{matrix} = H(\delta)|\, \delta - \hat{\delta} \qquad (2.27)$$

Where, H is the Hessian matrix,δ= (α , β , θ) and $\hat{\delta}$ =($\hat{\alpha}$, $\hat{\beta}$, $\hat{\theta}$). In order to maximize the probability function of the Observed Information Matrix, the Newton Raphson algorithm generates the variance-covariance matrix (2.27)

$$(-H(\delta)|\ \delta - \delta)^{-1} = \begin{pmatrix} var(\hat{\alpha}) & cov(\hat{\alpha}, \hat{\beta}) & cov(\hat{\alpha}, \hat{\theta}) \\ cov(\hat{\alpha}, \hat{\beta}) & var(\hat{\beta}) & cov(\hat{\beta}, \hat{\theta}) \\ cov(\hat{\alpha}, \hat{\theta}) & cov(\hat{\beta}, \hat{\theta}) & var(\hat{\theta}) \end{pmatrix} \quad (2.28)$$

Finally, we construct an approximate 100 (1-γ) %confidence interval for α , β and θ is

$$\hat{\alpha}^{+}_{-} = z_{y/2\sqrt{var(\hat{\alpha})}} ;\ \hat{\beta}^{+}_{-} = z_{y/2\sqrt{var(\hat{\beta})}} ;\ \hat{\theta}^{+}_{-} = z_{y/2\sqrt{var(\hat{\theta})}}$$

Where, $Z_{y/2}$ is the upper percentile of standard normal variate.

IV. Data analysis

4.1. Real DATA

This part analyzes a single real data set to validate our suggested model. In [2], genuine data from a test is examined. Deep groove ball bearings are extremely durable. The original discussion was by [19]. The data points are as follows: 17.88, 28.92, 33.0, 41.52, 42.12, 45.60, 48.40, 51.84, 51.96, 54.12, 55.56, 67.80, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04, and 173.4.

4.2. Parameter Estimation

We calculated parameter values by maximizing the log-likelihood function of our proposed model and five alternative Generalized Rayleigh (GR) models [17]. Exponentiated Chen (EC) [11], Exponentiated Exponential (EE) [22], Exponentiated Inverted Weibull (EIW) [13], and Compound Rayleigh (CR) [27] can be calculated directly using the optim() function in R software [9] and [23]. The Appendix displays the PDFs of the comparing models. Table 2 displays the MLE values for each parameter along with their standard errors.

MLE (SE)				
Models	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\vartheta}$
EE	1.02793(0.27632)	-	0.01516(0.00431)	-
EC	1.73629(0.79821)	0.47130(0.04631)	0.01517(0.00129)	
GR	1.01137(0.22086)	-	0.013421(0.001348)	-
CR	0.12457(0.024765)	1.76961(1.04831)	-	-
E|W	-	0.63781(0.06931)	-	12.1752(3.2671)
ERP	1.01776(0.21151)	0.53639(0.68952)	0.01347(0.00288)	-

4.3. Model Comparisons

We analyzed log-likelihood values and goodness of fit using Akaike's information criterion (AIC) and Bayesian information criterion (BIC), Corrected Akaike's Information Criterion (CAIC) and Hannan-Quinn Information Criterion (HQIC).

BIC = -2l(ϑ) + klog(n) ; CAIC= AIC+ $\frac{2k(k+1)}{n-k-1}$; and HQIC = -2l(ϑ)+2kln(ln(n))

The model's parameter count is denoted by k, and the sample size is represented by n.

AIC, BIC, CIAC, and HQIC report that the lowest the model with the highest value is the most effective among comparison models. The planned model has a lower value compared to the other models listed in the table. Therefore, our model is better than others.

Probability models	AIC	BIC	CAIC	HQIC	l($\boldsymbol{\vartheta}$)
EE	237.3243	241.634	236.9981	236.8715	-132.1875-
EC	228.4598	232.6391	228.1543	228.9318	-127.2583
ERP	219.3167	224.6526	220.7641	220.8714	-117.9718
GR	225.1986	228.0541	225.9863	225.7010	-121.0333
E|W	257.0071	259.8543	258.0642	258.1654	-139.8605
CR	289.3142	291.4289	288.9981	288.9175	-152.9080

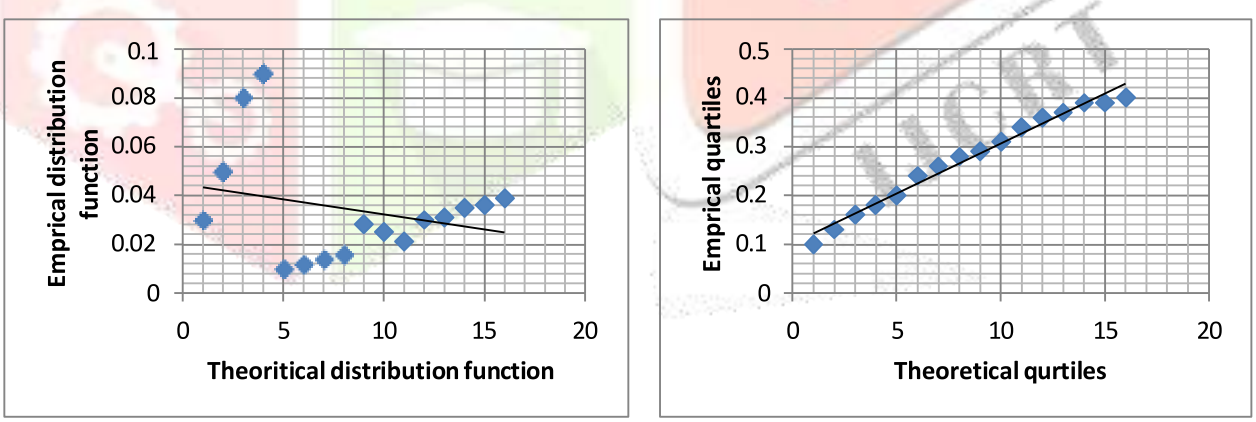
4.4. Model Validation

To confirm this result, examine the P-P and Q-Q graphs. PP and Q-Q graphs demonstrate the hypothesized distribution versus the empirical distribution. A P-P plot shows the points ($F(x_{(m)})$, $F(x_{(m)}; \delta)$) ; m= 1,2,…, i

Where, $\hat{\delta} = (\hat{\alpha}, \hat{\beta}, \hat{\theta})$ and x_m is the order statistics of the proposed model, of distribution function empirical and I(.) is a indicator function. Similarly the Q-Q plot has the points of

$$\left(x_m;\ F^{-1}\left(\frac{m}{j+1};\ \hat{\delta}\right)\right)$$

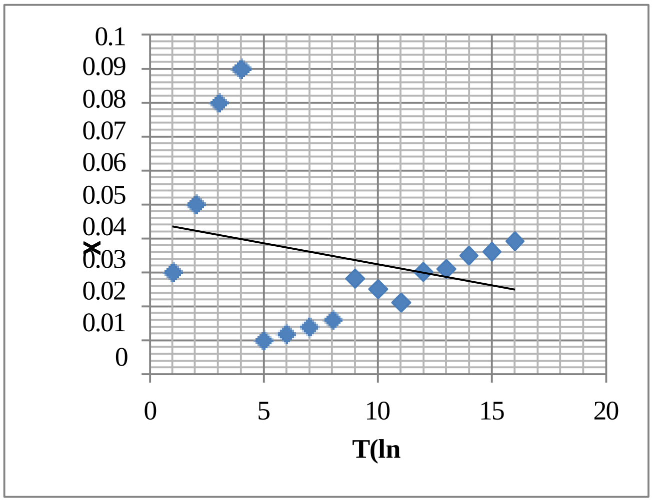
We observed that the model has good fit of theoretical distribution and empirical in both plots(Figure 3)

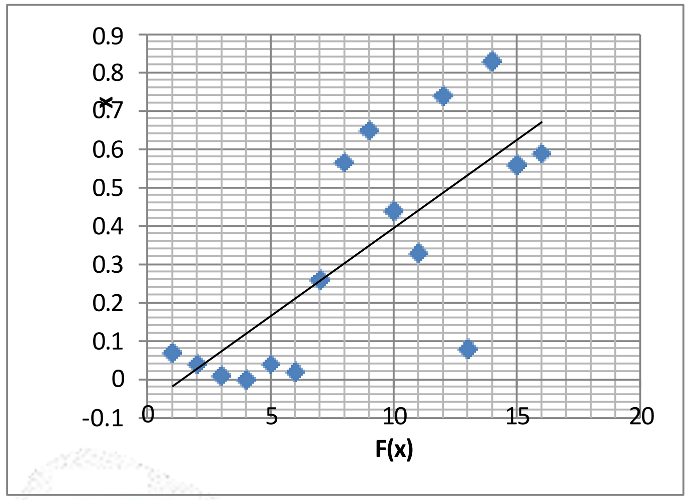


Moreover, the one-sample Kolmogorov-Smirnov test is validated. Given that the K-S test (D=0.14543) is not statistically significant, it appears to suit the comparison of the empirical and theoretical distributions (pvalue = 0.7155). It is evident from the plotting of the fitted distribution function and empirical distribution function in Figure 4 (left panel) that the suggested model fits the provided data satisfactorily. An essential graphical tool for determining whether or not our data set can be used to a specific model is the Total Time Test (TTT) plot. Because of [1], the field-based version of the TTT plot is given by

$$T\left(\frac{r}{j}\right) = \frac{\sum_{m=1}^{r} y_{mj} + (j-r)y_{mj}}{\sum_{m=1}^{r} y_{mj}}$$

Hence, the TTT plot of the data set is concave increasing in rate of the proposed distribution (Figure 4)





V. Conclusion

Our paper introduces the Exponentiated non-negative Rayleigh Poisson distribution, which belongs to the G family of distributions. Some mathematical characteristics of the new distribution, which includes probability weighted moments, order statistics, residual life function, quantile function, skewness, kurtosis, and survival function entropy, is well-derived. The hazard function displays an upward curve (concave) shape. Model comparison involves calculating AIC, BIC, CIAC, and HQIC. The proposed model outperforms previous models due to its lower values. The proposed model performs well when compared to other models in terms of CDF, EDF, and PDF. This technique can be applied to a variety of distributions for illustrative purposes, but we have chosen the Exponentiated Exponential Poisson G family model uses the Rayleigh distribution as its basis distribution. Several examples highlight the significance and adaptability of the new family. This study aims to serve as a reference for future research on the issue.

VI. Reference

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