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Comprehensive Governing Equations, Boundary Conditions And Nondimensional Parameters For Convective Instabilities

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Abstract: In this paper, we present a unified mathematical formulation of the governing equations relevant to convective instability problems in porous media, nanofluids, rotating convection, viscoelastic fluids, Maxwell fluids, and Oldroyd-B fluids. The basic equations, boundary conditions, and nondimensional parameters used throughout our Ph.D. research are clearly outlined. The model integrates the conservation laws of mass, momentum, energy, and species concentration under the Boussinesq approximation, incorporating buoyancy effects, viscous dissipation, Darcy–Brinkman resistance, anisotropic permeability, internal heat generation, Soret–Dufour diffusion, and non-Newtonian rheology where required. Key nondimensional groups including the Rayleigh, Darcy Rayleigh, Prandtl, Lewis, and Brinkman numbers are systematically defined.

This generalized framework provides a flexible foundation capable of representing a wide range of classical and modern convective instability configurations. It can be adapted to layered systems, rotating environments, oscillatory heating, chemically reactive mixtures, and nanofluid flows. The formulation is intended to serve as a reference for future studies, enabling rigorous stability analysis and facilitating comparisons across diverse physical systems.

Index Terms - Convective instability, Governing equations, Boundary conditions, Nondimensional parameters, Porous media.

1. INTRODUCTION:

Convective instability describes the transition of a fluid or a fluid-saturated porous medium from a stable conductive state to an active convective motion when buoyancy forces exceed diffusive effects. This process is central to many natural systems such as mantle flow, oceans, lakes, and geothermal reservoirs as well as engineered applications like reactors, insulation materials, heat pipes, and microfluidic devices.

Pure fluids are governed mainly by the Navier–Stokes equations, where inertia, viscosity, and diffusion interact. Porous media introduce additional complexity through drag, permeability, and anisotropy, requiring models such as Darcy, Brinkman, or Forchheimer formulations.

Many past studies treat only specific cases (e.g., thermal, double-diffusive, rotating, radiative, or chemically reacting convection), making it difficult to see the underlying unity. Modern applications including nanofluids, viscoelastic materials, and multi-component porous structures further demand a unified theoretical base.

This work builds such a framework by integrating classical conservation laws with buoyancy effects, porous drag terms, radiation, internal heating, and cross-diffusion mechanisms. Nondimensionalization yields key parameters like the Rayleigh and Darcy–Rayleigh numbers, revealing the balance between buoyancy, viscosity, diffusion, and advection.

Many authors and researchers have studied about the convective instability problems and their work (References) collectively provides a strong foundation for understanding convective instabilities in both fluid and porous media. Core texts by Nield & Bejan, Vafai, and Straughan establish the theoretical and mathematical framework for porous-media convection, while classical works by Chandrasekhar, Joseph, and Drazin & Reid lay out the fundamentals of hydrodynamic and stability theory. Foundational studies by Horton, Lapwood, and Elder introduce the early developments of Rayleigh–Darcy convection. Additional references address modern extensions such as nanofluids (Buongiorno, Rudyak), microscale flows (Kim & Karrila), double-diffusive and salt-finger convection (Bhatia & Steiner), and nonlinear or rotating porous-media behavior (Vadasz). Together, these sources form a comprehensive literature base for modeling, analyzing, and understanding convective processes in complex fluid and porous systems.

Overall, the model clarifies common principles across fluid and porous systems, shows how added physics modifies stability, and provides a versatile foundation for analyzing diverse convective instability problems.

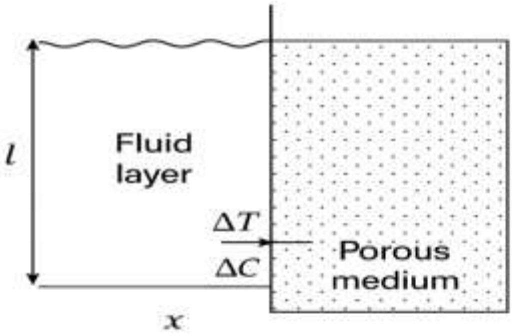


Fig. 1: Geometry of the coupled fluid–porous medium system

The left portion of the above Fig. 1 represents a clear fluid layer, while the right portion corresponds to a saturated porous medium. The interface between the two regions allows the exchange of heat and mass. Temperature and concentration differences, denoted by ΔT and ΔC, are imposed across the interface, creating buoyancy forces that may initiate instability in both regions. The vertical dimension indicates the depth of the system, and the dotted region visually distinguishes the porous medium. This configuration forms the foundation for analyzing how thermal and solutal gradients influence convective behavior in hybrid fluid -porous systems.

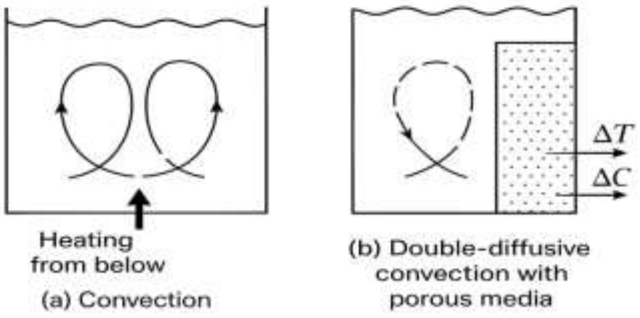


Fig. 2: a) Convection b) Double diffusive convection

The above Fig. 2a represents classical thermal convection in a fluid layer heated from below. The imposed temperature difference generates buoyancy forces, leading to overturning motions characterized by symmetric convection cells. These circulatory patterns emerge when the thermal gradient exceeds the critical threshold, marking the transition from a stable conductive state to an unstable convective regime.

The Fig. 2b extends the concept to a system influenced by both thermal and solutal gradients. Here, the fluid region and an adjacent porous layer experience differences in temperature (ΔT) and concentration (ΔC), leading to double-diffusive effects. The combination of heat and mass diffusion, which occur at different rates, can produce more complex instability patterns than pure thermal convection. The porous medium modifies the flow through drag and permeability, making the system suitable for studying coupled fluid–porous double-diffusive convection mechanisms.

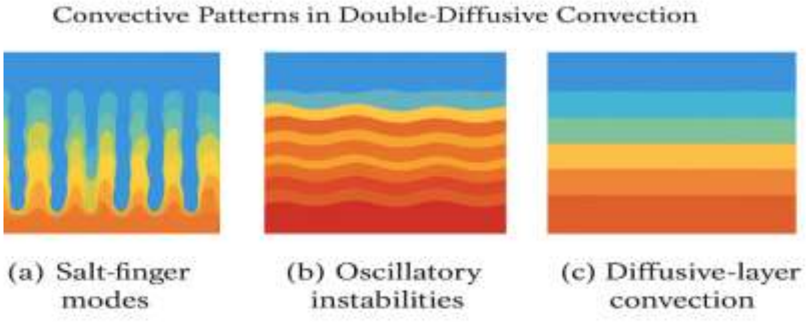


Fig. 3: Convective Patterns

The above Fig. 3 shows the convective patterns in double diffusive convection.

(a) Salt-finger modes:

- Occur in double-diffusive convection when a fluid has stabilizing temperature gradient and destabilizing salinity (or solute) gradient**.**
- The figure shows narrow vertical finger-like structures where heavier salty fluid sinks and lighter fresh fluid rises.
- Typical in oceans where warm salty water overlies cooler fresh water.

(b) Oscillatory instabilities:

- Arise when the system undergoes time-dependent convection**,** often due to competing effects of temperature and solute gradients.
- The figure shows waves or layers with periodic variations**,** indicating the fluid oscillates back and forth rather than forming steady convection cells.

(c) Diffusive-layer convection:

- Happens when solute diffuses slowly compared to heat**,** leading to stably stratified layers separated by thin convective zones.
- The figure shows horizontal layers with gradual transitions**,** representing regions where convection is confined to narrow layers.

2. Classification of Fluids:

Fluids can be classified in various ways depending on their flow behavior, composition, or physical properties**.** Here's a structured overview:

A. Based on Flow Behavior:

Newtonian Fluids:

- Definition: Fluids with constant viscosity regardless of shear rate.
- Examples**:** Water, air, gasoline, thin oils.
- Behavior: Linear relationship between shear stress and shear rate.

Non-Newtonian Fluids:

Definition: Fluids whose viscosity depends on shear rate or time.

Types:

- Shear-thinning (Pseudoplastic): Viscosity decreases with shear rate (e.g., ketchup, blood).
- Shear-thickening (Dilatant): Viscosity increases with shear rate (e.g., cornstarch in water).
- Bingham Plastic: Requires a yield stress to flow (e.g., toothpaste, mayonnaise).
- Thixotropic: Viscosity decreases over time under constant shear (e.g., gels, some paints).
- Rheopectic: Viscosity increases over time under constant shear (e.g., gypsum paste).

B. Based on Composition:

Single-phase Fluids:

- Liquids: Water, oil, alcohol.
- Gases: Air, nitrogen, steam.

Multi-phase Fluids:

- Suspensions: Solid particles in liquid (e.g., mud, slurry).
- Emulsions: Mixture of two immiscible liquids (e.g., milk, mayonnaise).
- Foams: Gas dispersed in a liquid or solid (e.g., whipped cream, shaving foam).
- Aerosols: Solid or liquid particles in gas (e.g., smoke, mist).

C. Based on Compressibility:

- Incompressible Fluids: Density remains almost constant (e.g., water, oil).
- Compressible Fluids: Density changes significantly with pressure (e.g., gases like air).

D. Based on Temperature Dependence:

- Ideal Fluids: No viscosity and incompressible (theoretical concept).
- Real Fluids: Have viscosity and may be compressible (all real liquids and gases).

E. Other Specialized Categories:

- Superfluids: Exhibit zero viscosity (e.g., liquid helium at very low temperatures).
- Viscoelastic Fluids: Show both viscous and elastic behavior (e.g., polymer solutions, blood).
- Magnetorheological Fluids: Change viscosity under magnetic field (e.g., smart fluids in dampers).

3. Navier–Stokes Equations:

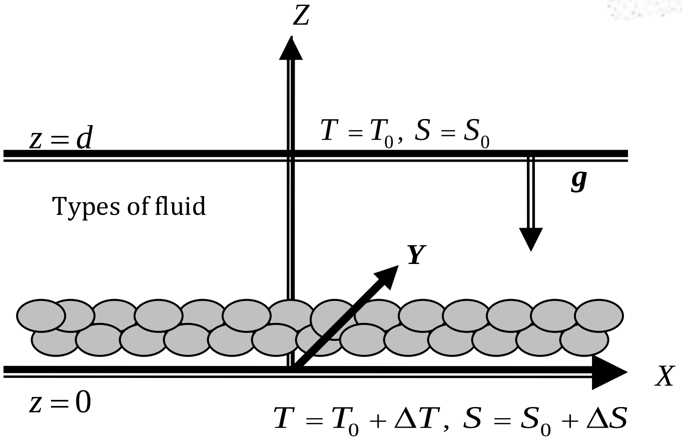
The Navier–Stokes equations are the fundamental mathematical equations that describe the motion of fluid substances—liquids, gases, and plasmas. They express how momentum**,** mass**,** and sometimes energy evolve within a fluid under the combined effects of viscous forces**,** pressure variations, and external body forces such as gravity or rotation.

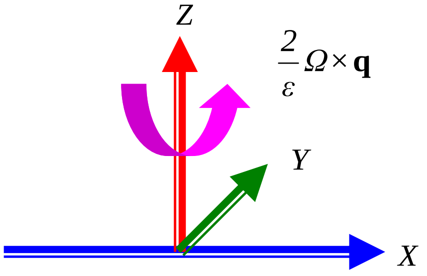
At their core, these equations result from applying:

1. Newton's second law (rate of change of momentum = sum of forces)
2. Conservation of mass (continuity equation)
3. Conservation of energy (optional depending on the problem)

Because fluids deform continuously and produce internal friction (viscosity), the Navier–Stokes equations include viscous diffusion terms, making them nonlinear partial differential equations that are often challenging to solve analytically.

4. **Geometry of the Problem of the Work:**





The above figures show the general physical configuration of the problems investigated in our thesis. We have considered an infinite horizontal (types of fluid) fluid saturated porous layer confined between two planes $z = 0$ and $z = d$, with the vertically downward gravity force $\boldsymbol{g}$ acting on it. The constant temperatures $\Delta T + T_0$ and T_0 with stabilizing concentrations $\Delta S + S_0$ and S_0 respectively are maintained between the lower and upper surfaces. A Cartesian frame of reference is chosen with the origin in the lower boundary and the z-axis vertically upwards. Concerned to the study we apply the external constraints like rotation, anisotropy , cross diffusion (Soret effect, Soret and Dufour effects) and internal heat source in both the Newtonian and non-Newtonian fluids to momentum, heat and mass equations with the Oberbeck – Boussinesq approximation, to modify and model the basic governing equations. The right side figure shows the physical configuration of convective study of rotational effects using same boundary conditions of left side figure.

After mathematical modeling of the problem, we solved the problems analytically using linear and nonlinear stability theory. For linear stability analysis we used the classical normal mode technique and for nonlinear analysis we have used the truncated Fourier series which provides the quantification of heat and mass transfer. The transient behavior of the Nusselt and Sherwood numbers is studied by solving numerically a fifth order Lorentz type system using the Runge-Kutta method.

5. Basic Equations (governing equations):

The central goal of fluid dynamics is to determine the velocity field in a given region, using the fundamental conservation laws of mass, momentum, and energy. In this work, we focus on these governing equations for fluids moving through porous media, including couple-stress, viscoelastic, and Maxwell fluids.

A porous medium consists of solid particles enclosing interconnected voids through which fluid flows under external forces. Only the connected, accessible voids contribute to the material’s porosity. Because the pore geometry is highly complex and microscopic, it is impossible to describe the exact flow in each pore or solve the full microscopic equations with precise boundary details.

To overcome this difficulty, researchers have long used the concept of a continuum equivalent, replacing the intricate pore structure with averaged properties over a representative volume. This approach has been widely applied in porous-media transport studies. In this study, we adopt this established microscopic-based averaging framework to analyze momentum and energy transport in saturated porous layers.

5.1 Continuity Equation (conservation of mass):

There are so many applications which depend on law of conservation of mass, for example, in river, circulatory system, respiratory system, air conditioning systems etc. The law of conservation of mass states that the amount of fluid flowing into a volume must be equal to the amount of fluid flowing out of that volume. The mathematical equivalence of the above statement is the so called, continuity equation,

$$\frac{\partial \rho}{\partial t}+\nabla\cdot\left(\rho\,\mathbf{q}\right)=0, \qquad (1)$$

where ρ and $\mathbf{q}$ are the density and velocity of the fluid respectively. Equation for flow through porous medium takes the form

$$\varepsilon\frac{\partial\rho}{\partial t}+\nabla\cdot\left(\rho\,\mathbf{q}\right)=0, \tag{2}$$

where ε is the porosity of a porous medium.

For an incompressible fluid the equation of continuity (1) and (2) reduces to

$$\nabla\cdot\mathbf{q}=0. \tag{3}$$

2. Momentum Equation (conservation of momentum):

(a) For flow through porous medium:

Darcy (1856) adopted the macroscopic approach and postulated that the flow in the porous media is directly proportional to the pressure gradient and body force and gave an empirical relation governed by

$$\mathbf{q}=\frac{\text{Constant}}{\mu}\;\left(-\nabla p+\rho\,\mathbf{g}\right). \tag{4}$$

Later Muskat (1937) has given a dynamical approach and proved that the proportionality constant in Eq. (4) should be related to the permeability of the porous material and it takes the form

$$\mathbf{q}=-\;\frac{K}{\mu}\;\left(\nabla p-\rho\,\mathbf{g}\right), \tag{5}$$

where μ is the fluid viscosity, K is the permeability of a porous medium, which determines the intrinsic properties of a medium and it has the dimension of length squared. Eq. (5) is known as Darcy's law for isotropic porous medium, which is still being used in the investigation of many practical problems because flow in this type of problems obeys potential nature of Darcy's law. Darcy's law explains about only a certain region of the porous flow process and it is applicable to only for a densely packed porous medium. This equation is valid when K is very small compared to a characteristic length.

The Darcy law (5) has been modified by including the local acceleration term, which is given by

$$\frac{\rho}{\varepsilon}\frac{\partial\mathbf{q}}{\partial t}=-\nabla p+\rho\,\mathbf{g}-\frac{\mu}{K}\,\mathbf{q}. \tag{6}$$

This equation is known as the **Darcy-Lapwood model.**

For the case of anisotropic porous medium, which possesses the horizontal isotropy, the Darcy-Lapwood Eq. (6) may be modified by incorporating the permeability tensor $\boldsymbol{K}=K_x^{-1}\left(\mathrm{ii}+\mathrm{jj}\right)+K_z^{-1}\left(\mathrm{kk}\right)$, in the form

$$\frac{\rho}{\varepsilon}\frac{\partial\mathbf{q}}{\partial t}=-\nabla p+\rho\,\mathbf{g}-\mu\,\mathbf{K}\cdot\mathbf{q}. \tag{7}$$

It is to be noted that for the case of isotropic porous medium we have taken the permeability $K_x=K_z=\boldsymbol{K}$, where $\boldsymbol{K}$ is the permeability tensor and K_x, K_y, K_z are the permeabilities in the x, y and z directions respectively.

There are many experimental and theoretical evidences, which suggest that the Darcy equation will sometime provide unsatisfactory explanation of the hydrodynamic conditions. It is well known that Darcy's law breaks down in situations where in other effects like viscous shear and inertia force come into action.

Beavers and Joseph (1967) and Beavers et al. (1970), Rajashekhar (1974), and Ranganna (1982) have experimentally demonstrated the existence of shear within the porous medium, near the boundaries, thus forming a region of shear influenced fluid flow. The Darcy equation cannot predict the existence of such a boundary region as no macroscopic shear term is included in the equation. Brinkman (1947), Tam (1969) and Rudraiah et al. (1979) have demonstrated that, to predict the boundary effects, the Brinkman (1947) equation of the form

$$\nabla p - \rho \mathbf{g} = -\frac{\mu}{K}\,\mathbf{q} + \mu_e\,\nabla^2 \mathbf{q}, \tag{8}$$

is the most suitable governing equation for an incompressible creeping flow of a Newtonian fluid within an isotropic and homogeneous porous medium. Eq. (8) is known as **Darcy-Brinkman model**. This equation contains two viscous terms on the right hand side. The first term is the usual Darcy term and the second is analogous to the Laplacian term that appears in the Navier-Stokes equation. The coefficient μ_e is the effective viscosity and μ is the fluid viscosity. Brinkman set μ and μ_e equal to each other, but in general they are only approximately equal. Therefore in the present study we have considered that ($\mu \neq \mu_e$).

The Brinkman Eq. (8) is physically consistent with experimentally observed boundary shear region on account of the usual viscous shear force $\mu_e \nabla^2 \mathbf{q}$. The Brinkman model is applicable for a sparsely packed fluid saturated porous medium in which there is more voids for a fluid to flow so that the distortions of velocity yields the usual viscous shear force.

In many practical problems, the flow through porous media is curvilinear and the curvature of the path yields the inertia effect, so that the streamlines become more distorted and the drag increases more rapidly. Lapwood (1948) was the first person who suggested for the inclusion of convective inertial term $(\mathbf{q}\cdot\nabla)\,\mathbf{q}$ in the momentum equation. Subsequently many research articles have been appeared on the non-Darcy model (Rudraiah 1983, Joseph et al. 1982, Nield 1983, Rudraiah 1984, Rudraiah 1997, Nield and Bejan 2006). Now with the usual inertia term $\frac{1}{\varepsilon^2}\,(\mathbf{q}\cdot\nabla)\,\mathbf{q}$ Eq. (8) may be written as

$$\frac{1}{\varepsilon^2}\,(\mathbf{q}\cdot\nabla)\,\mathbf{q} = -\nabla p + \rho\,\mathbf{g} - \frac{\mu}{K}\,\mathbf{q} + \mu_e \nabla^2 \mathbf{q}. \tag{9}$$

However, Eq. (9) does not take care of possible unsteady nature of velocity. The flow pattern in a certain region may be unsteady and one has to consider the local acceleration term $\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t}$ also. Adding this term to Eq. (9), then we have (9) implies

$$\rho\left[\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon^2}\,(\mathbf{q}\cdot\nabla)\,\mathbf{q}\right] = -\nabla p + \rho\,\mathbf{g} - \frac{\mu}{K}\mathbf{q} + \mu_e \nabla^2 \mathbf{q}, \tag{10}$$

this equation is known as **Darcy-Lapwood-Brinkman equation** (Rudraiah 1984). For anisotropic porous medium Eq. (10) takes the form

$$\rho\left[\frac{1}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \frac{1}{\varepsilon^2}\,(\mathbf{q}\cdot\nabla)\,\mathbf{q}\right] = -\nabla p + \rho\,\mathbf{g} - \mu\mathbf{K}\cdot\mathbf{q} + \mu_e\,\nabla^2 \mathbf{q}. \tag{11}$$

When the porous layer rotates about an axis parallel to the gravity vector there exist two types of forces. The first and important one is Coriolis force and the is centrifugal force. When we deal with the flow near the axis of rotation, we can ignore the effect of centrifugal force (Vadasz, 1998). Therefore, in such cases we must incorporate the Coriolis effect into the equation of motion, so as to explain the flow completely. The Darcy-Lapwood model must be extended to include the Coriolis term, $\frac{2}{\varepsilon}(\mathbf{\Omega}\times\mathbf{q})$ and neglecting the inertial term $\frac{1}{\varepsilon^2}(\mathbf{q}\cdot\nabla)\,\mathbf{q}$, in the form

$$\rho\left[\frac{1}{\varepsilon}\frac{\partial\mathbf{q}}{\partial t}+\frac{2}{\varepsilon}\mathbf{\Omega}\times\mathbf{q}\right]=-\nabla p+\rho\,\mathbf{g}-\frac{\mu}{K}\mathbf{q}\,. \tag{12}$$

This is the equation for the flow through a densely packed rotating porous layer.

For sparsely packed porous medium it is given by

$$\rho\left[\frac{1}{\varepsilon}\frac{\partial\mathbf{q}}{\partial t}+\frac{2}{\varepsilon}\Omega\times\mathbf{q}\right]=-\nabla p+\rho\mathbf{g}-\frac{\mu}{K}\mathbf{q}+\mu_e\nabla^2\mathbf{q}\,. \tag{13}$$

For horizontal sparsely packed anisotropic porous medium the momentum equation is given by

$$\rho\left[\frac{1}{\varepsilon}\frac{\partial\mathbf{q}}{\partial t}+\frac{2}{\varepsilon}\Omega\times\mathbf{q}\right]=-\nabla p+\rho\mathbf{g}-\mu K.\mathbf{q}+\mu_e\nabla^2\mathbf{q}\,. \tag{14}$$

(b) For the flow of clear viscous fluid:

The momentum equation for the viscous incompressible fluid is given by the usual Navier - Stokes equation

$$\rho\left[\frac{\partial\mathbf{q}}{\partial t}+(\mathbf{q}\cdot\nabla)\,\mathbf{q}\right]=-\nabla p+\rho\,\mathbf{g}+\mu\nabla^2\mathbf{q}\,. \tag{15}$$

(c) For the flow of couple stress fluid:

Number of theories has been proposed to explain the behaviour of fluids which contain microstructures such as additives, suspensions or granular matter. The theory of couple stress fluid given by Stokes (1966) defines the rotational field in terms of the velocity field, thereby reducing considerably the number of material constants in the constitutive equations characterizing fluid material. This theory introduces a second order gradient of velocity vector, instead of kinematically independent rotation vector in the constitutive relationship between stress and rate of stain.

Stokes theory of couple stress fluids is the simplest generalization of the classical theory of fluids which allows for the polar effects such as the presence of a non-symmetric stress tensor, couple stresses and body couples. The constitutive equations for couples stress fluids proposed by Stokes (1966) are

$$T_{(ij)}=(-p+\lambda D_{kk})\delta_{ij}+2\mu_f D_{ij}\,, \tag{16}$$

$$T_{[ij]}=-2\mu_1 W_{ij,kk}-\frac{\rho}{2}\varepsilon_{ijs}G_s\,, \tag{17}$$

$$M_{ij}=4\eta\mu_1\omega_{j,i}+4\mu_1^{'}\omega_{i,j}\,, \tag{18}$$

where $D_{ij}=\frac{1}{2}\left(\mathrm{q}_{i,j}+\mathrm{q}_{j,i}\right)$, $W_{i,j}=-\frac{1}{2}\left(\mathrm{q}_{i,j}-\mathrm{q}_{j,i}\right)$, $\omega_i=\frac{1}{2}\varepsilon_{ijk}\mathrm{q}_{k,j}$, $T_{(ij)}$ is the symmetric part and $T_{[ij]}$ is the anti-symmetric part of the stress tensor T_{ij}, M_{ij} the couple stress tensor, D_{ij} the deformation rate tensor,

W_{ij} the vorticity tensor, q_i the components of velocity vector, ω_i the vorticity vector, G_s the body couple, δ_{ij} the Kronecker delta, ρ the density, p the pressure, ε_{ijs} the alternating unit tensor, λ and μ_f are the material constants having the dimension of viscosity , μ_1 and μ_1' are the material constants having the dimension of momentum.

The ratio of (μ_1/μ_f) has the dimension of length squared and it characterizes the size of microstructure. In the flow of fluid with microstructure, the departure from classical theory depends on the relative size of substructure compared with the linear dimensions of the flow. In the case of incompressible fluids, when the body forces and body moments are absent, the momentum equation derived by Stokes (1966) in the vector notation become

$$\rho\left[\frac{\partial \mathbf{q}}{\partial t}+(\mathbf{q}\cdot\nabla)\,\mathbf{q}\right]=-\nabla p+\rho\,\mathbf{g}+\mu\nabla^2\mathbf{q}-\mu_1\,\nabla^4\mathbf{q}. \qquad (19)$$

This is the equation of motion for the couple stress fluid.

In case of polar fluids the action of one part of the body on its neighborhood cannot be represented by a force alone but rather by a force and couple. The last term on the right-hand side of Eq. (19) represents the effects of couple stresses in an incompressible fluid. Here μ_1 is a material constant responsible for the couple stress property and has the dimension of momentum (MLT^{-1}). According to Stokes' theory, the rheological flow properties for an incompressible viscous couple stress fluid are characterized by the two constants μ and μ_1. Since the dimension of μ is $ML^{-1}T^{-1}$ and that of μ_1 is MLT^{-1}, the ratio $(\mu_1/\mu)^{1/2}$ has the dimension of length.

The effects of couple stress are quite large for large values of the non-dimensional number l/d, where d is the typical dimension of the flow geometry and l is the material constant $l=(\mu_1/\mu)^{1/2}$. If l is a function of the molecular dimensions of the liquid, it will vary greatly for different liquids. For example, the length of a polymer chain may be a million times the diameter of water molecule. Therefore, one may expect that couple stresses appear in noticeable magnitudes in liquids with large molecules.

(d) For couple stress fluid flow through porous medium:

The momentum equation governing the motion of an incompressible couple stress fluid through a densely packed porous medium is given by

$$\frac{\rho}{\varepsilon}\left[\frac{\partial \mathbf{q}}{\partial t}+\frac{1}{\varepsilon}(\mathbf{q}\cdot\nabla)\,\mathbf{q}\right]=-\nabla p+\rho\,\mathbf{g}-\frac{1}{K}(\mu-\mu_c\nabla^2)\nabla^2\mathbf{q}. \qquad (20)$$

When the fluid permeates a porous material, the gross effect is represented by the Darcy's law. As a result of this microscopic law, the usual viscous and couple stress viscous terms in the momentum equation of couple stress fluid motion are replaced by the resistance term $-\frac{1}{K}(\mu-\mu_c\nabla^2)\mathbf{q}$ where μ and μ_c are the fluid viscosity and couple stress viscosity, and K is the permeability of the porous medium.

For incompressible couple stress fluid through a densely packed porous medium in the absence of body couple and inertia term is given by

$$\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} = -\nabla p + \rho\mathbf{g} - \frac{1}{K}\left(\mu - \mu_c\nabla^2\right)\mathbf{q}. \tag{21}$$

(e) For the flow of a Maxwell fluid through porous layer:

The equation of motion for the flow of Maxwell fluid through a porous medium is given by (Alisaev and Mirzadjanzade 1975, Khuzhayorov et al. 2000, and Tan and Masuoka 2005),

$$\left(1+\bar{\lambda}_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \nabla p - \rho\mathbf{g}\right) = -\frac{\mu}{K}\mathbf{q}\,. \tag{22}$$

We have not investigated the problem on anisotropic with this fluid but in that case of anisotropic porous medium Eq.(22) is given by

$$\left(1+\bar{\lambda}_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \nabla p - \rho\mathbf{g}\right) = -\mu\mathbf{K}\cdot\mathbf{q}. \tag{23}$$

(f) For the flow of viscoelastic fluid through porous layer:

For an Oldroyd-B fluid, the extra-stress tensor $\mathbf{T}$ is given by the constitutive equation (Tan and Masuoka, 2005)

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}\,,\ \mathbf{S} + \bar{\lambda}_1\frac{\mathbf{DS}}{\mathbf{D}t} = \mu\left(\mathbf{A} + \bar{\lambda}_2\frac{\mathbf{DA}}{\mathbf{D}t}\right), \tag{24}$$

where p is the hydrostatic pressure, $\mathbf{I}$ the identity tensor, μ the viscosity of the fluid, and $\mathbf{S}$ the extra stress tensor, $\bar{\lambda}_1$ and $\bar{\lambda}_2$ are constant relaxation and retardation times respectively. $\mathbf{A} = \nabla\mathbf{q} + \nabla\mathbf{q}^T$ is the strain-rate tensor, $\mathbf{q}$ is the velocity vector, ∇ is the gradient operator, and

$$\frac{\mathbf{DS}}{\mathbf{D}t} = \left(\frac{\partial}{\partial t} + \mathbf{q}\cdot\nabla\right)\mathbf{S} - \mathbf{S}(\nabla\mathbf{q}) - (\nabla\mathbf{q})^T\mathbf{S}\,, \tag{25}$$

$$\frac{\mathbf{DA}}{\mathbf{D}t} = \left(\frac{\partial}{\partial t} + \mathbf{q}\cdot\nabla\right)\mathbf{A} - \mathbf{A}(\nabla\mathbf{q}) - (\nabla\mathbf{q})^T\mathbf{A}. \tag{26}$$

It should be noted that this model includes the classical viscous Newtonian fluid as a special case for $\bar{\lambda}_1 = \bar{\lambda}_2 = 0$, and to be the Maxwell fluid when $\bar{\lambda}_2 = 0$.

It is well known that in flow of viscous Newtonian fluid at a low speed through a porous medium the pressure drop caused by the frictional drag is directly proportional to velocity, which is the Darcy's law. By analogy with Oldroyd-B constitutive relationships, the following phenomenological model, which relates pressure drop and velocity for a viscoelastic fluid in a porous medium has been given by (Khuzhayorov et al., 2000)

$$\left(1+\bar{\lambda}_1\frac{\partial}{\partial t}\right)\nabla p = -\frac{\mu}{K}\left(1+\bar{\lambda}_2\frac{\partial}{\partial t}\right)\mathbf{q}_D, \tag{27}$$

where K is permeability, $\mathbf{q}_D$ is Darcian velocity, which is related to the usual (i.e. volume averaged over a volume element consisting of fluid only in the pores) velocity vector $\mathbf{q}$ by $\mathbf{q}_D = \varepsilon\mathbf{q}$, ε is porosity of the porous medium. We note that when $\bar{\lambda}_1 = \bar{\lambda}_2 = 0$, Eq. (27) simplified to Darcy's law for flow of viscous Newtonian fluid through a porous medium. Thus Eq. (27) can be regarded as an approximate form of an empirical momentum equation for flow of Oldroyd-B fluid through a porous medium.

Under consideration of the balance of forces acting on a volume element of fluid, the local volume average balance of linear momentum is given by

$$\rho\frac{d\mathbf{q}}{dt} = -\nabla p + \rho\mathbf{g} + \nabla\cdot\mathbf{S} + \mathbf{r}\,, \tag{28}$$

where $\frac{d}{dt}$ is the material time derivative, $\mathbf{r}$ is Darcy resistance for an Oldroyd-B fluid in the porous medium. Since the pressure gradient in Eq. (28) can also be interpreted as a measure of the resistance to flow in the bulk of the porous medium, and $\mathbf{r}$ is a measure of the flow resistance offered by the solid matrix, thus $\mathbf{r}$ can be inferred from Eq. (28) to satisfy the following equation

$$\left(1+\bar{\lambda}_1\frac{\partial}{\partial t}\right)\mathbf{r} = -\frac{\mu\varepsilon}{K}\left(1+\bar{\lambda}_2\frac{\partial}{\partial t}\right)\mathbf{q}. \tag{29}$$

Substituting Eq. (28) into Eq. (29), we obtain

$$\left(1+\bar{\lambda}_1\frac{\partial}{\partial t}\right)\left(\rho\frac{d\mathbf{q}}{dt}+\nabla p-\rho\mathbf{g}-\nabla\cdot\mathbf{S}\right) = -\frac{\mu\varepsilon}{K}\left(1+\bar{\lambda}_2\frac{\partial}{\partial t}\right)\mathbf{q}. \tag{30}$$

For Darcy model, ignoring the advection term $\mathbf{q}\cdot\nabla\mathbf{q}$, and the viscous term $\nabla\cdot\mathbf{S}$, Eq. (30) can be simplified to (after dropping the suffix D on $\mathbf{q}$ for simplicity)

$$\left(1+\bar{\lambda}_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial\mathbf{q}}{\partial\mathbf{t}}+\nabla p-\rho\mathbf{g}\right) = -\frac{\mu}{K}\left(1+\bar{\lambda}_2\frac{\partial}{\partial t}\right)\mathbf{q}, \tag{31}$$

which is known as a modified Darcy-Oldroyd model. The equation of motion for the flow of Oldroyd-B fluid through a sparsely packed porous medium is given by (Vafai and Tien (1981), Tan and Masuoka (2005), and Malashetty et al. (2006)).

For sparsely packed porous medium Eq. (31) is given by

$$\left(1+\bar{\lambda}_1\frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial\mathbf{q}}{\partial t}+\nabla\, p-\rho\,\mathbf{g}\right) = \left(1+\bar{\lambda}_2\frac{\partial}{\partial t}\right)\left(-\frac{\mu}{K}\mathbf{q}+\mu_e\nabla^2\mathbf{q}\right). \tag{32}$$

5.3 Heat Transport Equation (conservation of thermal energy):

(a) For clear viscous fluid layer:

According to the Fourier second law of heat transfer the equation of heat transport for the viscous incompressible fluid, in the absence of heat source and viscous dissipation, is given by

$$\frac{\partial T}{\partial t} + (\mathbf{q}\cdot\nabla)\,T = \kappa_T\,\nabla^2 T, \tag{33}$$

where $\kappa_T = k/\rho c_p$ is the thermal diffusivity and c_p is the specific heat at constant pressure.

(b) For fluid-saturated porous layer:

Assuming thermal equilibrium between the fluid and solid phases and by ignoring the viscous dissipation and Darcy dissipation one can write a single-phase Fourier second law in the form:

$$\gamma \frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T = \kappa_T \nabla^2 T, \tag{34}$$

where $\gamma = \left[\varepsilon (\rho c)_f + (1-\varepsilon)(\rho c)_s \right] / (\rho c_p)_f$, is the specific heat ratio with the subscripts s and f denoting the quantity for solid porous matrix and fluid respectively.

For the case of thermal anisotropy the equation of energy is given by

$$\gamma \frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T = \nabla \cdot (\boldsymbol{\kappa}_{\mathbf{T}} \cdot \nabla T), \tag{35}$$

where $\boldsymbol{\kappa}_{\mathbf{T}} = \boldsymbol{\kappa}_{\mathbf{Tx}} (\mathrm{ii} + \mathrm{jj}) + \boldsymbol{\kappa}_{\mathbf{Tz}} (\mathrm{kk})$ is a thermal diffusivity tensor. Thermal isotropy is assumed in the horizontal plane.

In the presence of internal heat source (Q), Eq. (34) reduces to

$$\gamma \frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T = \kappa_T \nabla^2 T + Q(T - T_0). \tag{36}$$

5.4 Mass Transport Equation (conservation of mass species):

(a) For clear viscous fluid layer:

The term mass transfer is used here in a specialized sense, that is, the transport of a substance involved as component namely constituent and species in a fluid mixture (e.g., transport of salt in saline water). Convective mass transfer is analogous to convective heat transfer but when we deal with them separately new type of phenomena will arise.

Consider a fluid mixture of volume V and mass m. The total mass is equal to the sum of the individual masses m_i so that $m = \sum m_i$, where the subscript i refer to the i th component of the mixture. Hence the concentration of component i is defined as

$$S_i = \frac{m_i}{V}, \tag{37}$$

then the aggregate density ρ of the fluid mixture must be the sum of all the individual concentrations are defined as

$$\rho = \sum S_i . \tag{38}$$

Here, the unit of concentration is kgm^{-3}. Instead of S_i the alternative notation ρ_i is appropriate if one thinks of each component spread out over the total volume V.

We now apply the principle of conservation of mass to each component in the mixture. For the moment we use the notation ρ_i instead of S_i for the concentration of component i. In the absence of diffusion we must have

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{q}_i) = 0, \tag{39}$$

where $\mathbf{q_i}$ is the velocity of the particles of component i. Summing over i, we obtain

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \left(\sum \rho_i \mathbf{q}_i \right) = 0 . \tag{40}$$

This is same as

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{q}) = 0 \tag{41}$$

provided that we identify $\mathbf{q}$ with the averaged velocity as

$$\mathbf{q} = \frac{1}{\rho} \sum \rho_i \mathbf{q_i} . \tag{42}$$

Motion of component relative to this mass averaged velocity is called ‘diffusion’. Thus $(\mathbf{q_i} - \mathbf{q})$ is the diffusion velocity of component i and

$$\mathbf{j}_i = \rho_i (\mathbf{q_i} - \mathbf{q}) \tag{43}$$

is the ‘diffusive flux’ of the component i. Eq. (2.39) now becomes

$$\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i \mathbf{q}_i) = -\nabla \cdot \mathbf{j_i} . \tag{44}$$

Reverting to the notation S_i for concentration, and assuming that the mixture is incompressible, we have

$$\frac{\partial S_i}{\partial t} + (\mathbf{q} \cdot \nabla) S_i = -\nabla \cdot \mathbf{j_i} . \tag{45}$$

For the case of a two-component mixture, Fick’s law of mass diffusion is given by

$$\mathbf{j_1} = -D_{12} \nabla S_1 , \tag{46}$$

where D_{12} is the mass diffusivity of component 1 into component 2 and similarly for $\mathbf{j_2}$. In fact, $D_{12} = D_{21} = \kappa_S$, the solute (mass) diffusivity. The diffusivity depends on the mixture pressure, temperature and composition. Then Eqs. (45) and (46) yield

$$\frac{\partial S_1}{\partial t} + (q \cdot \nabla) S_1 = \nabla \cdot (\kappa_S \cdot \nabla S_1) . \tag{47}$$

If the migration of the first component is the only one of interest then the subscript can be dropped. For a homogeneous situation we have

$$\frac{\partial S}{\partial t} + (\mathbf{q} \cdot \nabla) S = \kappa_S \nabla^2 S \tag{48}$$

(b) For fluid-saturated porous layer:

$$\varepsilon \frac{\partial S}{\partial t} + (\mathbf{q} \cdot \nabla) S = \kappa_S \nabla^2 S . \tag{49}$$

where ε is the porosity, κ_S is the solute diffusivity.

5.5 Cross-Diffusions with Transport Equations:

In the case of cross-diffusion there exist two very important effects namely, the Soret effect and Dufour effect. The Soret effect describes the mass diffusion caused by temperature gradient, while the Dufour effect means the thermal diffusion caused by solute gradient. In such cases the equation of heat and mass transport need to modify, incorporating these two effects.

From a mathematical point of view, without loss of generality the heat and mass transport equations are written in the form

$$\gamma \frac{\partial T}{\partial t} + (\mathbf{q} \cdot \nabla) T = \kappa_T \nabla^2 T + \kappa_{TS} \nabla^2 S, \tag{50}$$

$$\varepsilon \frac{\partial S}{\partial t} + (\mathbf{q} \cdot \nabla) S = \kappa_S \nabla^2 S + \kappa_{ST} \nabla^2 T. \tag{51}$$

Here ε and γ are the porosity of porous medium and ratio of specific heat, respectively. Further, κ_T and κ_S are the thermal and solute diffusivities, κ_{ST} and κ_{TS} are the Soret and Dufour coefficients, respectively.

5.6 Equation of State:

The equation of state for a single component fluid can be derived by expanding the density $\rho(T)$ by Taylor's series about $T = T_0$ and doing so we get

$$\rho(T) = \rho(T_0) + (T - T_0)\left(\frac{\partial \rho}{\partial T}\right)_{T=T_0} + \frac{(T - T_0)^2}{2\,!}\left(\frac{\partial^2 \rho}{\partial T^2}\right)_{T=T_0} + ...$$

Neglecting the second and higher order terms, we get

$$\rho = \rho(T_0) + \left(\frac{\partial \rho}{\partial T}\right)_{T=T_0} (T - T_0).$$

By the definition of coefficient of thermal expansion

$$\beta_T = -\frac{1}{\rho_0}\left(\frac{\partial \rho}{\partial T}\right)_{T=T_0}.$$

Therefore,

$$\rho = \rho_0 [1 - \beta_T (T - T_0)]. \tag{52}$$

Similarly, the equation of state for two components system may given by

$$\rho = \rho_0 [1 - \beta_T (T - T_0) + s\, \beta_S (S - S_0)], \tag{53}$$

where ρ is the density, β_T, β_S are the coefficients of thermal and solute expansion respectively, T is the temperature field, and S is the concentration field, T_0, S_0 are the reference temperature and concentration fields respectively, and $s = \pm 1$.

6. Boussinesq Approximations:

In deriving the basic equations we have made the Boussinesq Approximations. There are many situations of practical occurrence in which the basic equations can be simplified considerably. These situations occur when the variability in the density and in the various coefficients is due to variations in temperature of only moderate amounts. The origin of simplifications in these cases is due to the smallness of the coefficient of volume expansion: for gases and liquids we shall be most concerned with, β is in the range of 10^{-3} to 10^{-4}. For variation in temperature not exceeding 10^{0}, the variations in density are at most 1 percent. The variations of this small amount can, in general, be ignored. But there is one important exception; variability of density in the term ρg in the equation of motion cannot be ignored; this is because the acceleration resulting from $\delta \rho g = \beta (\Delta T) g$ can be quite large; larger than the acceleration due to inertia term $(\mathbf{q}\cdot\nabla)\mathbf{q}$ in the equation of motion. Accordingly, we may treat density as constant in all terms in the equation of motion except the one in the external force.

The Boussinesq approximation is assumed to be valid, only when the speed of the fluid is much less than that of the sound (i.e. Mach number <<1) and all accelerations are slow compared with those associated with sound waves. The basic idea of this approximation is to filter out high frequency phenomenon such as sound waves since these are thought to be unimportant for transport processes.

The other important approximations which simplify significantly, the basic equations governing the problems considered in this study are as follows:

- The fluid properties namely kinematic viscosity, thermal diffusivity and the solute diffusivity are all assumed to be constants.
- Viscous dissipation, thermal dissipation and the Darcy dissipation are ignored.
- The effective viscosity, μ_e of the porous medium, is not the same as fluid viscosity, μ i.e., $\mu_e \neq \mu$.
- In case of anisotropic porous layer problems the medium is considered to be isotropic in both mechanical and thermal properties in the horizontal direction.
- The gravity acts vertically downwards.
- The axis of rotation of the channel is considered to parallel to the gravity vector.
- The Coriolis force is more important than the centrifugal force near the axis of rotation. Therefore, on can neglect the centrifugal acceleration in momentum equation.
- At a given instant, the fluid in the presence of solid matrix which is the sum of the stagnant thermal conductivity that is the temperature of fluid, T_f and the temperature of solid, T_S are the same i.e.,

  $$T_f = T_S = T .$$

7. Boundary Conditions:

The governing equations for the velocity and pressure fields are partial differential equations that are applicable at every point in a fluid that is being modeled as a continuum. To evaluate these, we need additional statements about the velocity field and possibly it’s gradient at the natural boundaries of the flow domain. Such statements are known as **boundary conditions**.

To solve the equations governing the problem under consideration we need suitable boundary conditions. The appropriate boundary conditions must be applied to both velocity and temperature field. The following boundary conditions have been used in the present study.

7.1 Velocity Boundary Conditions:

The boundary conditions on velocity are obtained from mass balance, the no-slip condition and the stress principle of Cauchy, depending upon the nature of the boundary surfaces of the fluid there appear different boundary conditions on velocity.

(a) Rigid surfaces:

If the flow configuration is bounded above and below by rigid surfaces, the no-slip condition is valid at the boundary surfaces and therefore we have

$$u = v = w = 0, \tag{54}$$

at the boundaries which are assumed to be at rest. In the study of the onset of thermal convection, it is common practice to assume the solution to be in the form of normal modes

$$\left(u,v,w\right)=w(z)\ \exp\left[i\left(lx+my\right)\right]. \tag{55}$$

Then the equation of continuity (2), using (54) and (55), gives

$$\frac{\partial w}{\partial z}=0 \text{ at the boundaries.} \tag{56}$$

Thus in the case of rigid boundaries, the boundary conditions on velocity are

$$w=\frac{\partial w}{\partial z}=0 \text{ at the boundaries.} \tag{57}$$

(b) Free surfaces:

When the channel is bounded below and above by free surfaces, the velocity at boundaries depends on whether surface tension is considered or not. If the free surface does not deform in the direction normal to itself, we must require

$$w=0 \quad \text{at the boundaries.} \tag{58}$$

In the absence of surface-tension, the free surface is free from shear stress so that

$$\frac{\partial u}{\partial z}=\frac{\partial v}{\partial z}=0 \quad \text{at the boundaries.} \tag{59}$$

From the equation of continuity, we have

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0. \tag{60}$$

Differentiating this with respect to ‘ z ’ and using Eq. (2.59), we get

$$\frac{\partial^2 w}{\partial z^2}=0 \text{ at the boundaries.} \tag{61}$$

Therefore, for free surfaces the appropriate boundary conditions on velocity are

$$w=\frac{\partial^2 w}{\partial z^2}=0 \quad \text{at the boundaries.} \tag{62}$$

7.2 Thermal Boundary Conditions:

The thermal boundary conditions to be applied at the upper and lower surfaces of the channel are based on the assumption that these surfaces are in contact with materials of infinite thermal conductivity and heat capacity. The thermal boundary conditions will depend on the nature of the boundaries. There are three kinds of thermal boundary conditions.

(a) Fixed surface temperature (isothermal):

When the boundary walls of the fluid layer have high heat conductivity and large heat capacity, then their temperature would be spatially uniform and unchanging in time. The boundary temperature in this case would be unperturbed by any flow or temperature perturbations in the fluid, thus

$$T=0 \text{ at the boundaries.} \tag{63}$$

These are known as isothermal boundary conditions or the boundary conditions of the first kind.

(b) Fixed surface heat flux (adiabatic):

When the bounding surfaces are free, there will be a heat exchange between the free surface and the environment. Then according to Fourier's law, the heat flux q_1 passing through the boundary per unit time and area is

$$q_1 = -k\frac{\partial T}{\partial z}\ , \tag{64}$$

where k is the thermal conductivity of the fluid and $\partial T/\partial z$ is the temperature gradient in the fluid at the boundary. Since there is exchange of heat between the system and the surrounding, q_1 remains unperturbed to thermal or flow perturbations in the fluid. Therefore, it follows from Eq. (64) that

$$\partial T/\partial z = 0, \tag{65}$$

at the boundaries. These are known as adiabatic or insulating boundary conditions or the thermal boundary conditions of the second kind.

(C) TEMPERATURE BOUNDARY CONDITIONS:

The thermal boundary conditions that are used in this study are the constant temperature boundary conditions i.e. isothermal conditions. The boundaries are held at constant different temperatures.

$$T = T_0 + \Delta T \quad \text{at} \quad z = 0 \tag{66}$$

$$T = T_0 \quad \text{at} \quad z = d \tag{67}$$

7.3 Solutal Boundary Conditions:

The solutal boundary conditions are analogous to those of thermal boundary conditions and we take $S =$ Constant at the boundaries therefore $S = 0$ is the boundary condition. This is known as permeable (isolated) boundary condition. For constant concentration flux at the boundaries, the boundary conditions are , $\frac{dS}{dz} = 0$ at the boundaries. (68)

This is known as impermeable boundary condition.

8. Nondimensionalization:

In addition to reducing the number of parameters, non-dimensionalized equation helps to gain a greater insight into the relative size of various terms present in the equation. Following appropriate selecting of scales for the non-dimensionalization process, this leads to identification of small terms in the equation. Neglecting the smaller terms against the bigger ones allows for the simplification of situation. Scaling helps provide better understanding of the physical situation, with the variation in dimensions of the parameters involved in the equation.

8.1 Non-Dimensional Parameters:

Non-dimensional analysis results in sound, orderly arrangement of the various physical quantities involved in problem. The magnitude of individual quantities encountered in a physical problem can be assembled into dimensionless groups, using dimensional analysis and the differential equations governing the fluid flow can be recast into non-dimensional forms. Such equations in the non-dimensional form always include one or more dimensionless groups. The dimensionless parameters, which appear in the resulting equations, are the parameters of solutions and are the key factors in determining the qualitative and quantitative nature of the flow phenomenon.

We have come across the following non-dimensional parameters during the course of investigation reported in this study.

- **Darcy number *(Da)*:**

The Darcy number is defined as, $Da = \frac{K}{d^2}$, physically, this represents the scale factor, which describes the extent of division of porous structure (permeability) as compared to the vertical extent of the porous layer. When the permeability is very high the resistance to the flow becomes effectively controlled by ordinary viscous resistance. In that case, the convection phenomenon is similar to that in an ordinary fluid layer. But in most of the problems, either the viscous force is negligible or is of comparable order to the Darcy resistance.

- **Thermal Rayleigh number (Ra_T):**

The thermal Rayleigh number is the ratio of buoyancy force to the viscous force. In case of a fluid layer it is given by $Ra_T = \frac{\beta_T g \Delta T d^3}{\nu\kappa}$, where ΔT is the temperature difference between the two boundaries, d is the thickness of the fluid layer, β_T is the thermal expansion coefficient, ν is the kinematics viscosity and κ is the thermal diffusivity.

Physically, the Rayleigh number signifies the relative importance of the buoyancy force to the dissipation force resulted from the viscous and thermal dissipation. Mathematically, this number denotes the Eigenvalue in the study of stability of thermal convection. Critical Rayleigh number is the value of the Eigenvalue at which the conduction state breaks and convection sets in.

For a fluid-saturated porous layer the Rayleigh number is to be modified by incorporating a Darcy number with it. Therefore, Rayleigh number is redefined as $Ra_D = \frac{\beta_T g \Delta T d K}{\nu\kappa}$. This is known as the Darcy-Rayleigh number.

- **Solute Rayleigh number** (Ra_S)**:**

Solute Rayleigh number is defined by taking the salinity gradient into account, $Ra_S = \dfrac{\beta_S g \Delta S d^3}{\nu\kappa}$, where β_S is the solute expansion coefficient, ΔS is the solute gradient maintained between the boundaries.

- **Soret parameter** (Sr)**:**

The Soret parameter is defined by $Sr = \dfrac{\kappa_{ST}\Delta T}{\kappa_T \Delta S}$, where κ_{ST} is the Soret coefficient, which quantifies the mass flux due to temperature gradient.

- **Dufour parameter** (Du)**:**

The Dufour parameter is defined by $Du = \dfrac{\kappa_{TS}\Delta S}{\kappa_T \Delta T}$, where κ_{TS} is the Dufour coefficient, which quantifies the heat flux due to concentration gradient.

- **Mechanical anisotropy parameter** (ξ)**:**

Mechanical anisotropy parameter is defined as $\xi = \dfrac{K_x}{K_z}$, where K_x, K_z are the permeability of the porous medium in horizontal and vertical directions respectively. Therefore, this non-dimensional parameter represents relative variation of the permeability in the horizontal and vertical directions.

- **Thermal anisotropy parameter** (η)**:**

Similar to the mechanical anisotropy parameter, one can define the thermal anisotropy parameter, by taking into account the relative variation of thermal properties in the horizontal and vertical directions, $\eta = \dfrac{\kappa_{Tx}}{\kappa_{Tz}}$, where κ_{Tx}, κ_{Tz} denote the thermal diffusivity in the horizontal and vertical directions respectively.

- **Lewis number** (Le)**:**

Lewis number is defined as the ratio of thermal diffusivity to the solute diffusivity and is given by $Le = \dfrac{\kappa_T}{\kappa_S}$ where κ_T is the thermal diffusivity, and κ_S is the solute diffusivity.

- **Diffusivity ratio** (τ)**:**

Diffusivity ratio is the ratio of solute diffusivity to the thermal diffusivity of the fluid and is given by $\tau = \dfrac{\kappa_S}{\kappa_T}$, where κ_S is the solute diffusivity and κ_T is the thermal diffusivity.

◆ Prandtl number (Pr):

Prandtl number is the ratio of kinematic viscosity to thermal diffusivity of the fluid and is given by $Pr = \frac{\nu}{\kappa_T}$, where '$\nu$' is the kinematic viscosity, κ_T is the thermal diffusivity. It depends on the intrinsic property of the fluid, not of a particular flow and plays a significant role especially under constrained systems.

In the case of porous medium the Darcy number (Da) is admitted into the effective Prandtl number to obtain a porous Prandtl number or the Darcy-Prandtl number.

$$Pr_D = \frac{\varepsilon \gamma Pr}{Da},$$

where γ is specific heat ratio, ε is the porosity, Pr is the Prandtl number, and Da is the Darcy number.

◆ Taylor number (Ta):

In case of a rotating channel, the effect of Coriolis force is of great importance and the relative significance of the Coriolis force and the viscous force is determined by a Taylor number. The Taylor number is the ratio of Coriolis force to the viscous force. It is defined by

$$Ta = \left(2\Omega d^2/\nu\right)^2, \text{ for fluid layer}$$

$$Ta = \left(2\Omega K/\varepsilon\nu\right)^2, \text{ for porous layer.}$$

Therefore, Ta represents the strength of the rotation of the fluid or porous layer and has a strong influence on the onset of convection.

◆ Couple stress parameter (C):

Couple stress parameter is defined as, $C = \frac{\mu_1}{\mu d^2}$, where μ_1 denotes the couple stress viscosity and is a material constant and other quantities have their usual meaning. Thus, couple stress parameter represents the extent of variation of the viscosity due to the presence of suspended particles.

◆ Vadasz number (Va):

Vadasz number is the ratio of the Prandtl number to the Darcy number and is given by $Va = \frac{\varepsilon \Pr}{Da}$, where $\Pr$ is the Prandtl number, Da is the Darcy number and ε is the porosity of the porous medium.

◆ Normalized porosity parameter (ε_n):

The normalized porosity parameter is expressed in terms of the porosity of the porous medium and the solid to fluid heat capacity ratio and is given by $\varepsilon_n = \frac{\varepsilon}{\gamma}$, where $\gamma = (\rho c)_m / \left(\rho c_p\right)_f$ is the ratio of specific heats, $(\rho c)_m = (1-\varepsilon)(\rho c)_s + \varepsilon\left(\rho c_p\right)_f$ where c_p is the specific heat of the fluid at constant pressure, c is the specific heat of the solid, the subscripts f, s and m denote fluid, solid and porous medium values, respectively.

- **Stress relaxation parameter (λ_1):**

The stress relaxation parameter is defined as $\lambda_1 = \frac{\kappa_T \bar{\lambda}_1}{\gamma d^2}$, where $\bar{\lambda}_1$ is the stress-relaxation time, i.e., a time required to decay the stress under constant deformation. Therefore, the stress relaxation time signifies importance of the viscous stress relaxation and thermal diffusion in the viscoelastic fluids.

- **Strain retardation parameter (λ_2):**

The strain retardation parameter is defined as $\lambda_2 = \frac{\kappa_T \bar{\lambda}_2}{\gamma d^2}$, where $\bar{\lambda}_2$ denotes the stain retardation time. Delayed elasticity is called retardation. Retardation is defined as the increase in deformation with time under constant stress.

- **Internal Rayleigh number (Ra_{int}):**

Internal Rayleigh number is defined as $Ra_{int} = \frac{Qd^2}{\kappa_T}$, where '$Q$' is the internal heat source, d is the height of the porous layer, and κ_T is the thermal diffusivity in the vertical direction.

- **Nusselt number (Nu):**

The non-dimensional coefficient of heat transfer is known as the Nusselt number and is given by

$$Nu = \frac{H}{\kappa_T \Delta T / d},$$ where H is the vertical heat flux.

Physically, this represents the heat transport by both conduction and convection. The variation of Nu with Rayleigh number is called the heat transport curve. Nu remains equal to unity up to the critical Rayleigh number but increases above unity with the onset of convection.

- **Sherwood number (Sh):**

The transport of mass due to conduction and convection is given by solute Nusselt number. The non-dimensional coefficient of mass transfer is known as the solute Nusselt number. It is given by

$$Sh = \frac{J}{\kappa_S \Delta S / d},$$ where J is the vertical mass flux.

9. Conclusion:

A comprehensive mathematical model for convective instabilities in fluids and porous media has been developed, incorporating Newtonian and non-Newtonian behaviors, porous structures, rotation, double-diffusive effects, thermal modulation, chemical reactions, and nanoparticle-induced variations. The model provides a unified framework for linear and nonlinear stability analysis, identifying key nondimensional parameters that govern the onset and nature of convection, including oscillatory and salt-finger modes. Porous media effects, viscous and Darcy dissipation, and rotational influences are shown to significantly alter convective thresholds and pattern formation. This framework offers predictive capability for complex convective phenomena, providing a foundation for theoretical, numerical, and experimental studies in engineering and geophysical applications. Future extensions could include heterogeneous porous structures and magnetohydrodynamic effects to enhance real-world applicability.

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