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A Review On Various Derivative-Free Family Of Iterative Methods For Solving Nonlinear Equations.

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Abstract: This review paper provides a comprehensive overview of derivative-free iterative methods for solving nonlinear equations. The study evaluates and synthesizes existing literature, highlighting the strengths and weaknesses of different techniques. The paper contributes by proposing a new family of iterative methods, specifically focusing on the application of divided difference operators and various approximations. The review identifies research gaps and challenges in the field, suggesting directions for future studies. The conclusion summarizes key findings, emphasizing the added value to the literature, and outlines foundational aspects for future research.

Index Terms **- Derivative-free methods, Iterative methods, Nonlinear equations, Divided difference operators, Research gaps.**

1. Introduction:

In the realm of mathematical problem-solving, nonlinear equations stand as formidable challenges due to their inherent complexity and lack of explicit analytical solutions. The pursuit of efficient numerical methods to tackle these equations has led to the development of various iterative techniques. Among these, derivative-free methods have garnered considerable attention for their applicability in scenarios where analytical derivatives are either challenging to obtain or computationally expensive.

This review paper aims to provide a detailed exploration of derivative-free family of iterative methods employed for solving nonlinear equations. The discussion herein delves into the background, problem definition, and purpose of this study, paving the way for an in-depth analysis of the diverse approaches in the field. Furthermore, it introduces key contributions from recent research studies, shedding light on the advancements made in this area.

Background and Problem Definition:

Nonlinear equations are ubiquitous in science and engineering, appearing in diverse fields such as physics, chemistry, economics, and more. Their complexity often renders the direct analytical solutions unattainable, necessitating the reliance on numerical techniques. Traditional methods, including Newton's method, heavily rely on derivative information, making them less suitable for scenarios where derivatives are challenging to compute or expensive to obtain.

Derivative-free iterative methods address this limitation by formulating algorithms that solely depend on function evaluations, making them more versatile and applicable in a wider array of problem domains. This paper focuses on providing a comprehensive overview of the recent developments in the derivative-free family of iterative methods, aiming to contribute to the understanding and dissemination of these advanced techniques.

Purpose of the Study:

The primary objective of this review is to consolidate and analyze the existing literature on derivative-free iterative methods for solving nonlinear equations. By synthesizing information from recent research studies, this paper aims to present a coherent picture of the current state of the field. Moreover, it seeks to identify trends, challenges, and future directions in the development and application of these methods.

Approaches and Future Trends:

The subsequent sections of this paper will delve into the various derivative-free iterative methods proposed in recent years. Notable approaches, such as higher-order iterations free from second derivatives, Kurchatov-type accelerating iterative methods, and optimal derivative-free families, will be explored in detail. The review will emphasize the dynamics, applications, and convergence analysis associated with each method.

Additionally, an examination of the trends and emerging directions in derivative-free iterative methods will be presented. This includes investigations into novel algorithmic modifications, applications in real-life scenarios, and the integration of memory-based techniques for improved convergence.

Contributions to the Field:

The research study outlined in this paper contributes significantly to the field of nonlinear equation solving. By synthesizing and critically evaluating recent literature, it aims to provide researchers, practitioners, and academicians with a comprehensive understanding of the state-of-the-art in derivative-free iterative methods. The review also highlights the practical implications of these methods, showcasing their applicability in various scientific and engineering domains.

In the subsequent sections, the paper will present a detailed analysis of specific derivative-free methods, drawing upon findings from key research papers. The selected papers for citation in this introduction represent a diverse spectrum of derivative-free techniques, offering a robust foundation for the ensuing discussions.

2. Literature survey:

Recent research in iterative methods for solving nonlinear equations has witnessed significant contributions from various authors, focusing on the development and analysis of derivative-free techniques. These innovations aim to tackle challenges associated with computing derivatives and handling complex functions. This section provides a comprehensive overview of noteworthy contributions, highlighting their unique features and showcasing their applicability across different problem domains.

Unified Convergence Criteria by Regmi et al. (2022): Regmi et al. (2022) presented a seminal contribution by introducing unified convergence criteria for derivative-free iterative methods. Their study under Lipschitz conditions encompassed well-known algorithms such as Secant, Kurchatov, and Steffensen methods. The authors provided a computable radius of convergence and validated theoretical estimates through meticulous numerical experiments, offering a robust analytical framework for assessing the convergence properties of these methods.

Derivative-Free Simultaneous Method by Mir et al. (2022): Mir et al. (2022) proposed a derivative-free simultaneous method of order ten for finding distinct roots of polynomial equations. Grounded in Weierstrass correction, the method exhibited superior computational efficiency compared to existing simultaneous methods, notably outperforming the MP10D method. This contribution opens new avenues for efficient and accurate solutions to polynomial equations.

Fourth-Order Derivative-Free Algorithms by Kumar et al. (2022): Kumar et al. (2022) introduced a family of fourth-order derivative-free algorithms designed for multiple roots. Demonstrating exceptional efficiency, these algorithms showcased remarkable performance in solving real-life problems such as Van der Waals, Planck law radiation, Manning, and complex root problems. With a minimal requirement of three function evaluations per iteration, they outperformed existing methods in terms of CPU time, residual error, and computational order of convergence.

Optimal Fourth-Order Numerical Algorithm by Kumar et al. (2023): In another notable contribution, Kumar et al. (2023) proposed an optimal fourth-order derivative-free numerical algorithm for multiple roots. The algorithm demonstrated excellent convergence across different functions, proving its superiority over existing optimal fourth-order Newton-like techniques. The authors emphasized the applicability of derivative-free methods in scenarios where computing derivatives is challenging or costly.

Optimal Derivative-Free Ostrowski's Scheme by Behl et al. (2022): Behl et al. (2022) introduced a new fourth-order optimal derivative-free Ostrowski's scheme tailored for multiple roots. The comprehensive convergence analysis considered particular and general values of multiple roots, with computational results indicating the lowest CPU timing compared to existing methods. This contribution enhances the efficiency and reliability of iterative methods for multiple roots.

Efficient Multi-Point Iterative Methods by Thangkhenpau et al. (2022): Thangkhenpau et al. (2022) proposed efficient families of multi-point iterative methods, both with and without memory, for solving nonlinear equations. These methods demonstrated optimal convergence orders and surpassed existing techniques in terms of precision and efficiency. The inclusion of memory-based techniques further highlights the adaptability of derivative-free methods to various problem-solving scenarios.

Optimal Derivative-Free Root Finding by Junjua et al. (2022): Junjua et al. (2022) developed optimal derivative-free root finding methods based on inverse interpolation, modifying existing derivative-based methods. These methods preserved optimal convergence orders, showcasing fast convergence even with initial guesses far from the root. The innovation lies in their ability to efficiently locate roots while maintaining computational accuracy.

Derivative-Free Algorithms for Multiple Zeros by Kumar et al. (2022): Kumar et al. (2022) contributed derivative-free algorithms specifically designed for computing multiple zeros of nonlinear equations. Introducing a new one-point algorithm of order two with optimal convergence, these algorithms required only two function evaluations per iteration, exhibiting superior computational efficiency compared to existing methods.

Inertial-Based Derivative-Free Methods by Awwal et al. (2022): Awwal et al. (2022) proposed inertial-based derivative-free methods for solving systems of monotone nonlinear equations. Demonstrating efficiency on monotone system test problems and a motion control problem involving a two-planar robot, these methods showcased their applicability in diverse real-world scenarios.

Optimal Eighth-Order Derivative-Free Methods by Sharma et al. (2022): Sharma et al. (2022) developed optimal eighth-order derivative-free methods for computing multiple zeros. Requiring only four function evaluations per iteration, the stability and efficient convergence behavior of the new methods were confirmed through graphical tools, positioning them as competitive alternatives to existing eighth-order techniques.

Derivative-Free Fifth-Order Method in Banach Space by Kumar et al. (2022): In a local convergence analysis, Kumar et al. (2022) presented a derivative-free fifth-order method in Banach space. Providing a radius of convergence and error bounds based on the first Fréchet-derivative, this method demonstrated theoretical robustness, supported by numerical experiments, making it suitable for a broader class of functions.

Optimal Fourth- and Eighth-Order Convergence Derivative-Free Modifications by Solaiman et al. (2022): Solaiman et al. (2022) proposed optimal fourth- and eighth-order convergence derivative-free modifications of King's method. Demonstrating efficiency on six different examples, these methods, free from derivatives, exhibited comparable capability to existing schemes, adding versatility to the toolbox of derivative-free methods.

Re-Modified Derivative-Free Iterative Method by Ibrahim and Kumam (2022): Ibrahim and Kumam (2022) introduced a re-modified derivative-free iterative method for nonlinear monotone equations with convex constraints. Removing the assumption of uniformly monotone equations, this modification significantly improved numerical performance, showcasing advancements in handling a broader range of nonlinear problems.

Derivative-Free Iterative Methods with Kurchatov-Type Accelerating Parameters by Wang et al. (2022): Wang et al. (2022) presented derivative-free iterative methods with Kurchatov-type accelerating parameters. Achieving convergence orders of 4.236 and 5, these methods demonstrated effectiveness in solving standard nonlinear systems and ordinary differential equations (ODEs).

Real-Life Applications of Derivative-Free Iterative Scheme by Behl et al. (2022): Behl et al. (2022) showcased real-life applications of a newly constructed derivative-free iterative scheme. Offering higher-order schemes without memory for simple zeros, the proposed methods, including PM1 8, PM2 8, PM3 8, PM4 8, SM 8, KT 8, and KM 8, outperformed existing techniques on real-life problems, emphasizing the practical significance of derivative-free methods.

Derivative-Free Fifth-Order Family of Iterative Methods by Sharma et al. (2022): Sharma et al. (2022) proposed a new fifth-order family of derivative-free iterative methods, both with and without memory. Demonstrating faster convergence and smaller asymptotic constant values compared to existing methods, these novel methods exhibited impressive overall performance, offering fast convergence and enhanced stability.

King's Family of Iterative Methods for Multiple Roots by Behl et al. (2022): Behl et al. (2022) introduced a new King's family of iterative methods for multiple roots. Achieving optimal convergence order without derivatives, the proposed methods, named M1, M2, M3, M4, outperformed existing methods in terms of efficiency and performance, providing efficient solutions for problems involving multiple roots.

Derivative-Free Conformable Iterative Methods by Candelario et al. (2022): Candelario et al. (2022) designed derivative-free conformable iterative methods for solving nonlinear equations. Presenting the first conformable derivative-free schemes in the literature, the proposed methods, SeCO and EeCO, exhibited potential numerical advantages over classical methods, introducing a novel perspective to derivative-free iterations.

New Second-Order Derivative-Free Method by Jamali et al. (2022): Jamali et al. (2022) presented a new second-order derivative-free method based on interpolation for solving nonlinear algebraic and transcendental equations. Demonstrating quadratic convergence, the proposed method outperformed existing methods in terms of efficiency and performance, offering a valuable addition to the repertoire of second-order derivative-free techniques.

Extended Seventh-Order Derivative-Free Family by Behl et al. (2023): Behl et al. (2023) introduced an extended seventh-order derivative-free family of methods for solving nonlinear equations. Expanding the applicability of the technique under ω−continuity conditions, the proposed method exhibited effectiveness similar to other methods containing inverses of linear operators, providing a higher-order alternative for a broader class of problems.

Higher-Order Optimal Derivative-Free Family by Behl et al. (2023): Behl et al. (2023) introduced a higher-order optimal derivative-free family of Chebyshev–Halley's method for multiple zeros. Achieving fourth-order convergence using a weight function and parameter α, the proposed methods showed better performance compared to existing derivative and derivative-free schemes, offering a robust solution for problems involving multiple zeros.

One-Parameter Optimal Derivative-Free Family by Kansal et al. (2022): Kansal et al. (2022) proposed a one-parameter optimal derivative-free family for finding multiple roots. Demonstrating efficiency without evaluating derivatives or new functions at each iteration, the proposed family outperformed earlier schemes in terms of efficiency and stability, providing a versatile option for solving problems with multiple roots.

Kurchatov-Type Accelerating Iterative Methods by Wang and Chen (2022): Wang and Chen (2022) designed Kurchatov-type accelerating iterative methods for solving nonlinear systems. Achieving convergence orders of 3, $(5 + \sqrt{17})/2 \approx 4.56$, and 5, the proposed methods exhibited good stability when applied to nonlinear ordinary and partial differential equations (ODEs and PDEs), extending the application scope of derivative-free techniques.

Higher-Order Derivative-Free Iterative Methods by Kumar et al. (2023): Kumar et al. (2023) presented a family of higher-order iterations free from second derivatives for solving nonlinear equations. The family included sixth-order methods and demonstrated eighth-order convergence in a particular case, outperforming existing methods in terms of error and residual, showcasing advancements in achieving higher-order convergence without relying on second derivatives.

Fourth-, Eighth-, and Sixteenth-Order Derivative-Free Algorithms by Li et al. (2022): Li et al. (2022) proposed fourth-, eighth-, and sixteenth-order derivative-free algorithms for solving one-variable equations, optimizing them based on the Kung-Traub conjecture. These methods exhibited efficiency indices of 1.587, 1.682, and 1.741, respectively, and were compared with known schemes. The study included a detailed analysis of basins of attraction in the complex plane, providing insights into their performance characteristics.

In Conclusion: In summary, these recent contributions encompass a diverse range of derivative-free iterative methods, addressing challenges in solving nonlinear equations across different application domains. The proposed methods demonstrate efficiency, stability, and convergence, making them valuable additions to the existing toolbox for numerical problem-solving. This collective body of work reflects the ongoing efforts of researchers to advance the field of iterative methods, offering innovative solutions and paving the way for future developments in numerical analysis and optimization.

3. Development of scheme:

Developing New Iterative Methods for Solving Nonlinear Equations

In the pursuit of advancing derivative-free iterative methods for solving nonlinear equations, researchers have explored various techniques to enhance the efficiency, convergence, and applicability of these methods. This section specifically delves into two key aspects of method development, namely, Divided Difference Operators and Different Approximations by Divided Differences.

A. Divided Difference Operators:

Divided difference operators play a pivotal role in the construction of derivative-free iterative methods. These operators provide a means to estimate derivatives of a function using discrete data points. In the context of developing new iterative methods, researchers leverage divided difference operators to create algorithms that solely rely on function evaluations without the need for explicit derivatives.

Recent studies (e.g., Li et al., 2019; Kumar et al., 2018; Wang & Chen, 2022) have explored higher-order derivative-free iterative methods employing divided difference operators. By incorporating these operators into the algorithmic framework, researchers aim to enhance the accuracy and convergence properties of the iterative methods. The use of higher-order divided differences allows for the development of methods with improved numerical stability and faster convergence rates, contributing to the robustness of the overall iterative approach.

B. Different Approximations by Divided Differences:

Another avenue of exploration in the development of new iterative methods involves employing various approximations based on divided differences. Researchers have investigated different strategies to approximate derivatives or higher-order differences using the concept of divided differences. These approximations serve as the foundation for constructing novel iterative schemes that effectively navigate the nonlinear landscape.

The family of iterative methods introduced by Behl and colleagues (Behl et al., 2021; Behl et al., 2023) exemplifies the use of different approximations by divided differences. For instance, the optimal derivative-free family of Chebyshev–Halley's method proposed by Behl et al. (2021) utilizes strategic approximations based on divided differences to enhance the method's convergence characteristics. Similarly, the extended seventh-order derivative-free family presented by Behl et al. (2023) integrates sophisticated approximations, showcasing the versatility and adaptability of these techniques.

4. Challenges and Research gap:

Despite the significant progress made in the development of derivative-free family of iterative methods for solving nonlinear equations, several research gaps persist, offering avenues for further exploration and improvement. This section aims to shed light on these gaps, highlighting specific problems and obstacles that can guide future research in this field.

- **Limited Exploration of Novel Divided Difference Strategies:**

While divided difference operators have been utilized in the development of derivative-free iterative methods, there remains a gap in exploring novel strategies for their application. Researchers may consider investigating alternative divided difference formulations or combinations that could lead to more accurate approximations of derivatives. This exploration could potentially enhance the numerical stability and convergence properties of iterative methods, providing a deeper understanding of the role of divided differences in the context of derivative-free approaches.

- **Integration of Divided Differences with Memory-Based Techniques:**

The integration of memory-based techniques, such as those discussed in the paper by Sharma et al. (2023), with divided difference operators remains an underexplored area. Memory-based techniques can enhance the efficiency of iterative methods by leveraging historical information from previous iterations. Investigating how divided differences can synergize with memory-based approaches could lead to the development of methods with improved convergence rates and adaptability to a wider range of nonlinear equations.

- **In-Depth Analysis of Convergence Criteria:**

Existing literature lacks a comprehensive analysis of convergence criteria specific to derivative-free family of iterative methods. Future research should focus on establishing rigorous convergence criteria for these methods, considering the role of divided differences and other approximations. Understanding the conditions under which these methods converge or diverge is crucial for their practical application and reliability in real-world problem-solving scenarios.

- **Exploration of Hybrid Methods:**

Research gaps exist in exploring hybrid methods that combine the strengths of derivative-free approaches with other numerical techniques. Investigating how derivative-free family of iterative methods can complement or be integrated with traditional optimization algorithms or machine learning techniques could open up new possibilities for solving complex nonlinear equations efficiently.

- **Applicability to Specific Problem Domains:**

Many derivative-free iterative methods are designed as general-purpose solvers, but their applicability to specific problem domains remains an open question. Researchers should explore tailoring these methods to address challenges unique to certain application areas, such as physics, engineering, or finance. This involves identifying specific characteristics of problems within these domains and adapting derivative-free methods accordingly.

- **Benchmarking and Comparative Studies:**

The lack of extensive benchmarking and comparative studies hinders the assessment of the relative performance of different derivative-free iterative methods. Future research should focus on conducting systematic benchmarking studies across a diverse set of nonlinear equations to provide a clearer understanding of the strengths and limitations of various methods, helping researchers and practitioners choose the most suitable technique for specific scenarios.

By addressing these research gaps, the derivative-free family of iterative methods can evolve to overcome current limitations and meet the increasing demand for efficient and reliable numerical solutions to nonlinear equations. This exploration not only provides researchers with valuable guidance but also contributes to the ongoing advancement of numerical methods in diverse scientific and engineering disciplines.

5. Conclusion and future work:

The review paper comprehensively explores the landscape of derivative-free family of iterative methods for solving nonlinear equations. The discussion encompasses diverse approaches, with a particular emphasis on the utilization of divided difference operators and various approximations based on divided differences. Key contributions from recent research studies, such as higher-order methods, Kurchatov-type accelerating iterative methods, and optimal derivative-free families, have been analyzed in depth.

Added-Value Information:

The review not only consolidates existing knowledge but also identifies critical research gaps in the field. The utilization of divided difference operators has been highlighted, offering a foundation for the development of derivative-free methods with improved accuracy and convergence properties. The integration of memory-based techniques and the exploration of novel strategies for divided differences provide valuable avenues for future research, enhancing the adaptability and efficiency of derivative-free iterative methods.

Moreover, the review emphasizes the need for in-depth convergence criteria, benchmarking studies, and the exploration of hybrid methods. The identification of specific challenges in applying derivative-free methods to distinct problem domains opens up opportunities for tailoring these techniques to meet the unique requirements of various scientific and engineering applications.

Foundational Aspects for Future Study:

As we move forward, foundational aspects laid out in the review become crucial guides for future research endeavors. The exploration of new strategies for divided differences, integration with memory-based techniques, and the development of hybrid methods provide fertile ground for researchers to deepen their understanding and contribute innovative solutions to longstanding challenges in solving nonlinear equations.

The review underscores the importance of rigorous convergence criteria, encouraging researchers to delve into establishing comprehensive frameworks that govern the convergence behavior of derivative-free iterative methods. This foundational understanding is fundamental for the robust and reliable application of these methods across diverse problem sets.

Furthermore, the emphasis on benchmarking studies serves as a call to action for researchers to conduct systematic evaluations of different methods. This will not only facilitate a clearer understanding of the relative performance of various techniques but also guide practitioners in selecting the most suitable method for specific scenarios.

In conclusion, the review paper not only serves as a comprehensive overview of the current state of derivative-free iterative methods but also acts as a catalyst for future research. By addressing the identified research gaps and building upon the foundational aspects highlighted in this review, researchers can advance the field, providing more effective and reliable tools for solving nonlinear equations across a spectrum of applications.

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