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ON WEAKLY $\mathcal{G}\Omega$-CLOSED SETS IN TOPOLOGICAL SPACES

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Abstract: In this paper, we introduce a new class of generalized closed sets called $\mathcal{G}\Omega$-closed and weakly $\mathcal{G}\Omega$-closed sets. Also, we investigate the relationships among related generalized closed sets.

Keywords: $\mathcal{G}\Omega$-closed, w$\mathcal{G}\Omega$ -closed, gsp-closed, gs-closed, sg-closed.

1. INTRODUCTION

Levine [11] introduced generalized closed sets in general topology as a generalization of closed sets. This idea was viewed as valuable and many outcomes overall generalised topology were gotten to the next level. Many researchers like Veerakumar [19] introduced $\hat{g}$-closed sets in topological spaces. Sheik John [17] introduced ω-closed sets in topological spaces. After the coming of these ideas, generalised topologists presented different sorts of generalised closed sets and concentrated on their major properties. As of late, Ravi and Ganesan [16] presented and $\ddot{g}$ -closed sets in general topolgy as one more generalization of closed sets and demonstrated that the class of $\ddot{g}$ -closed sets appropriately lies between the class of closed sets and the class of g-closed sets.

Pious Missier et al. [15] have presented the idea of g'''-closed sets and concentrated on their most basic properties in topological spaces.

In this paper, we present class of generalized closed sets called weakly $\mathcal{G}\Omega$-closed sets. Also, Additionally, we examine the connections among related generalized closed sets.

2. PRELIMINARIES

Throughout this thesis (X, τ), (Y, σ) and (Z, η) (or X, Y and Z) represent topological spaces (briefly $\mathcal{TPS}$) on which no separation axioms are assumed unless otherwise mentioned. For a subset $\mathcal{P}$ of a space X, cl($\mathcal{P}$), int($\mathcal{P}$) and $\mathcal{P}^c$ or X| $\mathcal{P}$ or X – $\mathcal{P}$ denote the closure of $\mathcal{P}$, the interior of $\mathcal{P}$ and the complement of $\mathcal{P}$, respectively.

We recall the following definitions which are useful in the sequel.

Definition 2.1

A subset $\mathcal{P}$ of a space X is called:

(i) semi-open [10] if $\mathcal{P} \subseteq$ cl(int($\mathcal{P}$));

(ii) α-open [13] if $\mathcal{P} \subseteq$ int(cl(int($\mathcal{P}$)));

(iii) semi-preopen [1] if $\mathcal{P} \subseteq$ cl(int(cl($\mathcal{P}$)));

(iv) regular open [18] if $\mathcal{P}$ = int(cl($\mathcal{P}$)).

The complements of the above mentioned open sets are called their respective closed sets.

Definition 2.2

A subset $\mathcal{P}$ of a space X is called:

(i) a generalized closed (briefly g-cld) [11] if cl($\mathcal{P}$) $\subseteq \mathcal{B}$ whenever $\mathcal{P} \subseteq \mathcal{B}$ and $\mathcal{B}$ is open in X.

(ii) a generalized semiclosed (briefly gs-cld) [3] if scl($\mathcal{P}$) $\subseteq \mathcal{B}$ whenever $\mathcal{P} \subseteq \mathcal{B}$ and $\mathcal{B}$ is open in X.

(iii) a α-generalized closed (briefly α g-cld) set [12] if α cl($\mathcal{P}$) $\subseteq \mathcal{B}$ whenever $\mathcal{P} \subseteq \mathcal{B}$ and $\mathcal{B}$ is open in X.

(iv) a generalized semi-preclosed (briefly gsp-cld) set [9] if spcl($\mathcal{P}$) $\subseteq \mathcal{B}$ whenever $\mathcal{P} \subseteq \mathcal{B}$ and $\mathcal{B}$ is open in X

(v) a semi-generalized closed (briefly sg-cld) [5] if scl($\mathcal{P}$) $\subseteq \mathcal{B}$ whenever $\mathcal{P} \subseteq \mathcal{B}$ and $\mathcal{B}$ is semi-open in X.

3. WEAKLY $\mathcal{G}\Omega$ -CLOSED SETS

We introduce the definition of weakly $\mathcal{G}\Omega$-closed sets in $\mathcal{TPS}$ and study the relationships of such sets.

Definition 3.1 A subset $\mathcal{P}$ of a $\mathcal{TPS}$ is called

(i) a $\mathcal{G}\Omega$ -closed (briefly $\mathcal{G}\Omega$ -cld) if cl($\mathcal{P}$) $\subseteq \mathcal{B}$ whenever $\mathcal{P} \subseteq \mathcal{B}$ and $\mathcal{B}$ is sg-open in X.

(ii) a weakly $\mathcal{G}\Omega$-closed (briefly w$\mathcal{G}\Omega$-cld) if cl(int($\mathcal{P}$)) $\subseteq \mathcal{B}$ whenever $\mathcal{P} \subseteq \mathcal{B}$ and $\mathcal{B}$ is sg-open in X.

Theorem 3.2

Any closed is w$\mathcal{G}\Omega$-cld but converse is not true.

Proof

Let $\mathcal{P}$ be a closed set such that $\mathcal{P} \subseteq \mathcal{B}$. Then cl($\mathcal{P}$) = $\mathcal{P}$. Let $\mathcal{B}$ be sg-open. Since int($\mathcal{P}$) $\subseteq \mathcal{P}$, cl(int($\mathcal{P}$)) $\subseteq$ cl($\mathcal{P}$) = $\mathcal{P}$. We have cl(int($\mathcal{P}$)) $\subseteq \mathcal{P} \subseteq \mathcal{B}$ whenever $\mathcal{P} \subseteq \mathcal{B}$ and $\mathcal{B}$ is sg-open. Thus $\mathcal{P}$ is w$\mathcal{G}\Omega$-cld.

Example 3.3

Let X = {i, s, d} and τ = {ϕ, {i}, {s}, {i, s}, X}. Then the set {i, s} is w$\mathcal{G}\Omega$-cld set but not closed in X.

Theorem 3.4

Any $\mathcal{G}\Omega$-cld is w$\mathcal{G}\Omega$-cld but converse is not true.

Proof

Let $\mathcal{P}$ be a $\mathcal{G}\Omega$-closed. Then by definition cl($\mathcal{P}$) $\subseteq \mathcal{B}$ whenever $\mathcal{P} \subseteq \mathcal{B}$ and $\mathcal{B}$ is sg-open in X. Since int($\mathcal{P}$) $\subseteq$ $\mathcal{P}$, cl(int($\mathcal{P}$)) $\subseteq$ cl($\mathcal{P}$) = $\mathcal{P}$. We have cl(int($\mathcal{P}$)) $\subseteq \mathcal{P} \subseteq \mathcal{B}$ whenever $\mathcal{P} \subseteq \mathcal{B}$ and $\mathcal{B}$ is sg-open. Hence $\mathcal{P}$ is w$\mathcal{G}\Omega$-cld.

Example 3.5

Let X = {i,s,d} and τ = {ϕ, {i},{s},{i, s}, X}. The set {i,s} is w$\mathcal{G}\Omega$-cld set but not $\mathcal{G}\Omega$-cld in X.

Theorem 3.6

Any regular cld is w$\mathcal{G}\Omega$-cld but converse is not true.

Proof

Let $\mathscr{P}$ be any regular closed set .Then by definition $\mathscr{P}$ = cl(int($\mathscr{P}$)) $\subseteq \mathscr{B}$. Let $\mathscr{B}$ be sg-open set containing $\mathscr{P}$. Hence, $\mathscr{P}$ is w$\mathcal{G}\Omega$-cld.

Example 3.7

Let X = {i,s,d} and τ = {ϕ, {i},{s},{i, s}, X}, the set {a} is w$\mathcal{G}\Omega$-cld but not regular closed in X.

Theorem 3.8

Any w$\mathcal{G}\Omega$-cld is gsp-cld but converse is not true.

Proof

Let $\mathscr{P}$ be any w$\mathcal{G}\Omega$-cld and $\mathscr{P} \subseteq \mathscr{B}$ and $\mathscr{B}$ be open set. Then $\mathscr{B}$ is a sg-open set containing $\mathscr{P}$ and cl(int($\mathscr{P}$)) $\subseteq \mathscr{B}$. Hence int(cl(int($\mathscr{P}$))) $\subseteq$ int $\mathscr{B} = \mathscr{B}$, Since $\mathscr{B}$ is open which shows that spcl($\mathscr{P}$) = $\mathscr{P} \cup$ int(cl(int(,$\mathscr{P}$))) $\subseteq$ U. Thus, $\mathscr{P}$ is gsp-closed.

Example 3.9

Let X = {i, s, d} and τ = {ϕ, {i}, {s}, {i, s}, X}. Then the set {i} is gsp-cld but not w$\mathcal{G}\Omega$-cld.

Theorem 3.10

If a subset $\mathscr{P}$ of a $\mathscr{TPS}$ X is both closed and α g-cld, then it is w$\mathcal{G}\Omega$-cld in X.

Proof

Let $\mathscr{P}$ be an α g-closed set in (X, τ) and $\mathscr{B}$ be open set containing $\mathscr{P}$. Then by definition $\mathscr{B} \supseteq \alpha$ cl($\mathscr{P}$) = $\mathscr{P} \cup$ cl(int(cl($\mathscr{P}$))) whenever $\mathscr{P} \subseteq \mathscr{B}$ and $\mathscr{B}$ is an open set. Since $\mathscr{P}$ is closed, $\mathscr{B} \supseteq$ cl(int($\mathscr{P}$)) and hence $\mathscr{P}$ is w$\mathcal{G}\Omega$-cld in X.

Theorem 3.11

If a subset $\mathscr{P}$ of a $\mathscr{TPS}$ X is both open and w$\mathcal{G}\Omega$-cld, then it is closed.

Proof

Since $\mathscr{P}$ is open, int $\mathscr{P}$= $\mathscr{P}$ and $\mathscr{P}$ is sg open. Since $\mathscr{P}$ is w$\mathcal{G}\Omega$-cld, cl($\mathscr{P}$)= cl(int($\mathscr{P}$)) $\subseteq \mathscr{P}$. Also $\mathscr{P} \subseteq$ cl($\mathscr{P}$). which implies that int($\mathscr{P}$) $\subseteq$ int(cl($\mathscr{P}$)). Hence $\mathscr{P}$ is closed in (X, τ).

Corollary 3.12

If a subset $\mathscr{P}$ of a $\mathscr{TPS}$ X is both open and w$\mathcal{G}\Omega$-cld, then it is both regular open and regular cld in X.

Proof

Since $\mathscr{P}$ is open, int $\mathscr{P} = \mathscr{P}$ and $\mathscr{P}$ is sg open. Since w$\mathcal{G}\Omega$-cld, $cl(\mathscr{P}) = cl(int(\mathscr{P})) \subseteq \mathscr{P}$, $int(cl(\mathscr{P})) \subseteq int(\mathscr{P}) = \mathscr{P}$. Also we have $\mathscr{P} \subseteq cl(\mathscr{P})$, $\mathscr{P} = int(\mathscr{P}) \subseteq int(cl(\mathscr{P}))$, which implies that $\mathscr{P} = int(cl(\mathscr{P}))$. Hence $\mathscr{P}$ is regular open.

Since $cl(\mathscr{P}) \subseteq \mathscr{P}$, $cl(int\mathscr{P}) \subseteq \mathscr{P}$. Also $\mathscr{P} \subseteq cl(\mathscr{P}) = cl(int\ \mathscr{P})$. Which shows that $\mathscr{P} \subseteq cl(int\mathscr{P})$. Hence $\mathscr{P}$ is regular closed.

Theorem 3.13

Suppose that $B \subseteq \mathscr{P} \subseteq X$, B is a sg-cld relative to $\mathscr{P}$ and that $\mathscr{P}$ is both open and sg-cld subset of X. Then B is sg-cld relative to X.

Proof

Let $\mathscr{B}$ be a sg-closed set such that $\mathscr{B} \subseteq O$, O is open in X. Then $\mathscr{B} \subseteq \mathscr{P} \cap O$ and $scl(\mathscr{B}) \subseteq \mathscr{P} \cap O$. It follows that $\mathscr{P} \cap scl(\mathscr{B}) \subseteq \mathscr{P} \cap O$ and $\mathscr{P} \subseteq O \cup (scl(\mathscr{B}))^c$. Since $\mathscr{P}$ is sg-closed in X, we have $scl(\mathscr{P}) \subseteq O \cup (scl(\mathscr{B}))^c$ since the union of open set and semi-open set is semi-open. Therefore $scl(\mathscr{B}) \subseteq scl(\mathscr{P}) \subseteq O \cup (scl(\mathscr{B}))^c$ and consequently, $scl(\mathscr{B}) \subseteq O$.

Corollary 3.14

Let $\mathscr{P}$ be both open and sg-cld and suppose that F is closed. Then $\mathscr{P} \cap F$ is sg-cld.

Proof

Since $\mathscr{P}$ is both open and sg-cld and F is closed. Also we have $\mathscr{P} \cap F \subseteq \mathscr{P}$. Thus $\mathscr{P} \cap F$ is closed in $\mathscr{P}$ and hence by theorem 1.3.13 $\mathscr{P} \cap F$ is sg-cld in $\mathscr{P}$.

Theorem 3.15

A set $\mathscr{P}$ is w$\mathcal{G}\Omega$-cld if and only if $cl(int(\mathscr{P})) - \mathscr{P}$ contains no non-empty sg-cld.

Proof

Necessity. Let us assume that F be a sg-cld such that $F \subseteq cl(int(\mathscr{P})) - \mathscr{P}$. Since F^c is sg-open and $\mathscr{P} \subseteq F^c$, by definition of w$\mathcal{G}\Omega$-cld it follows that $cl(int(\mathscr{P})) \subseteq F^c$. ie. $F \subseteq (cl(int(\mathscr{P})))^c$. This implies that $F \subseteq (cl(int(\mathscr{P}))) \cap (cl(int(\mathscr{P})))^c = \phi$.

Sufficiency. Conversely Assume that there exist no non-empty sg closed set contained in cl(int($\mathscr{P}$)) $-\mathscr{P}$. Let G be both closed and sg-open set in X such that $\mathscr{P} \subseteq$ G. If $\mathscr{P}$ is not w$\mathcal{G}\Omega$-cld, then cl(int($\mathscr{P}$)) is not contained in G, then cl(int($\mathscr{P}$)) $\cap$ G^c is a non-empty sg-closed subset of cl(int($\mathscr{P}$)) $-$ $\mathscr{P}$, which is a contradiction. Hence $\mathscr{P}$ is w$\mathcal{G}\Omega$-cld.

Theorem 3.16

Let X be a $\mathcal{TPS}$ and $\mathscr{P} \subseteq$ Y $\subseteq$ X. If $\mathscr{P}$ is open and w$\mathcal{G}\Omega$-closed in X, then $\mathscr{P}$ is w$\mathcal{G}\Omega$-cld relative to Y.

Proof

Let $\mathscr{B}$ is sg-open in (X, τ) such that $\mathscr{P} \subseteq \mathscr{B} \cap$ Y. Since $\mathscr{P}$ is w$\mathcal{G}\Omega$-cld in X, $\mathscr{P} \subseteq \mathscr{B}$ implies cl(int($\mathscr{P}$)) $\subseteq \mathscr{B}$. That is Y $\cap$ (cl(int($\mathscr{P}$))) $\subseteq$ Y $\cap$ $\mathscr{B}$ where Y $\cap$ cl(int($\mathscr{P}$)) is closure of interior of $\mathscr{P}$ in Y. Thus, $\mathscr{P}$ is w$\mathcal{G}\Omega$-cld relative to Y.

Theorem 3.17

If a subset $\mathscr{P}$ of a $\mathcal{TPS}$ X is nowhere dense, then it is w$\mathcal{G}\Omega$-cld.

Proof

Since int($\mathscr{P}$) $\subseteq$ int(cl($\mathscr{P}$)) and $\mathscr{P}$ is nowhere dense, int($\mathscr{P}$) = ϕ. Therefore cl(int($\mathscr{P}$)) = ϕ and hence $\mathscr{P}$ is w$\mathcal{G}\Omega$-cld in X.

The converse of Theorem 3.17 need not be true as seen in the following example.

Example 3.18

Let X = {i, s, d} and τ = {ϕ, {i}, {s, d}, X}. Then the set {i} is w$\mathcal{G}\Omega$-cld but not nowhere dense in X.

Remark 3.19

The following examples show that w$\mathcal{G}\Omega$-cld and semi-closedness are independent.

Example 3.20

Let X = {i, s, d} and τ = {ϕ, {i}, {s, d}, X}. Then the set {s} is w$\mathcal{G}\Omega$-cld but not semi-cld in X.

Example 3.21

Let X = {i, s, d} and τ = {ϕ, {i}, {s}, {i, s}, X}. Then the set {s} is semi-closed set but not w$\mathcal{G}\Omega$-cld in X.

Definition 3.22

A subset $\mathscr{P}$ of a $\mathscr{TPS}$ X is called w$\mathscr{G}\Omega$-open set if A^c is w$\mathscr{G}\Omega$-cld in X.

Theorem 3.23

Any open set is w$\mathscr{G}\Omega$-open.

Proof

Let $\mathscr{P}$ be an open set in a $\mathscr{TPS}$ X. Then $\mathscr{P}^c$ is closed in X. By Theorem 3.2 it follows that $\mathscr{P}^c$ is w$\mathscr{G}\Omega$-cld in X. Hence $\mathscr{P}$ is w$\mathscr{G}\Omega$-open in X.

The converse of Theorem 3.23 need not be true as seen in the following example.

Example 3.24

Let X = {i,s,d} and τ = {ϕ, {i},{s},{i, s}, X}.Then the set {d} is w$\mathscr{G}\Omega$-open set but it is not open in X.

Proposition 3.25

(i) Every $\mathscr{G}\Omega$-open set is w$\mathscr{G}\Omega$-open but not conversely.

(ii) Every regular open is w$\mathscr{G}\Omega$-open but not conversely.

(iii) Every w$\mathscr{G}\Omega$-open set is gsp-open but not conversely.

Proof

(i) Let $\mathscr{P}$ be an $\mathscr{G}\Omega$-open set in a topological space X. Then $\mathscr{G}\Omega$-^{c} is closed in X. By Theorem 1.3.4 it follows that $\mathscr{G}\Omega$-^{c} is w$\mathscr{G}\Omega$-cld in X. Hence $\mathscr{G}\Omega$- is w$\mathscr{G}\Omega$-open in X.

(ii) Let $\mathscr{P}$ be an regular open set in a topological space X. Then $\mathscr{P}^c$ is regular closed in X. By Theorem 1.3.6 it follows that $\mathscr{P}^c$ is w$\mathscr{G}\Omega$-cld in X. Hence $\mathscr{P}$ is w$\mathscr{G}\Omega$-open in X.

(iii) Let $\mathscr{P}$ be an w$\mathscr{G}\Omega$-open set in a topological space X. Then $\mathscr{P}^c$ is w$\mathscr{G}\Omega$ closed in X. By Theorem 1.3.8 it follows that $\mathscr{P}^c$ is gsp-cld in X. Hence $\mathscr{P}$ is gsp open in X.

Theorem 3.26

A subset $\mathscr{P}$ of a $\mathscr{TPS}$ X is w$\mathscr{G}\Omega$-open if $G \subseteq int(cl(\mathscr{P}))$ whenever $G \subseteq \mathscr{P}$ and G is sg-cld.

Proof

Let $\mathscr{P}$ be any w$\mathcal{G}\Omega$-open. Then $\mathscr{P}^{c}$ is w$\mathcal{G}\Omega$-cld. Let G be a sg-cld contained in $\mathscr{P}$. Then G^{c} is a sg-open set containing $\mathscr{P}^{c}$. Since $\mathscr{P}^{c}$ is w$\mathcal{G}\Omega$-cld, we have $cl(int(\mathscr{P}^{c})) \subseteq G^{c}$. Therefore $G \subseteq int(cl(\mathscr{P}))$.

Conversely, we suppose that $G \subseteq int(cl(\mathscr{P}))$ whenever $G \subseteq \mathscr{P}$ and G is sg-closed. Then G^{c} is a sg-open set containing $\mathscr{P}^{c}$ and $G^{c} \supseteq (int(cl(\mathscr{P})))^{c}$. It follows that $G^{c} \supseteq cl(int(\mathscr{P}^{c}))$. Hence $\mathscr{P}^{c}$ is w$\mathcal{G}\Omega$-cld and so $\mathscr{P}$ is w$\mathcal{G}\Omega$-open.

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