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On Strongly Sgp-Continuous And Perfectly Sgp-Continuous Functions In Topological Spaces

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Abstract: In this paper, we introduce a new class continuous functions called strongly sgp-continuous functions and perfectly sgp-continuous by using sgp-continuous functions in topological spaces. We observed that completely sgp-continuous functions are strongly sgp-continuous and discuss the characterizations of this class and study its properties and investigate the relationships and their properties.

***Index Terms* - sgp-closed sets ,sgp-open sets, strongly sgp-continuous functions, perfectly sgp-continuous functions, completely sgp-continuous functions ect..**

Introduction: : The notions of semi-open sets were introduced and studied by Levine [5] in 1963. Njastad [14] and Mashhour et al [10] introduced and studied The concept of generalized closed (briefly g-closed) sets were introduced and studied by Levine [6] In 1970, Levine [6] initiated the study of generalized closed(g-closed) sets, that is , a subset A of a topological space X is g-closed if the closure of A included in every open superset of A and defined a T1/2 space to be one in which the closed sets and g-closed sets coincide. The notion has been studied extensively in recent years by many topologists. In 1982, Malghan [9] introduced and studied the concept of generalized closed functions. After that several topologists introduced and studied α-open functions, gp-closed functions, gs-closed functions and gpr-closed functions . The concept of homeomorphism has been generalized by many topologists. Crossley and Hildebrand [2] have introduced and studied semihomeomorphisms which are strictly weaker than homeomorphisms in topological spaces. Maki et al [7] have introduced and studied g-homeomorphisms and gc-homeomorphisms in topological spaces. Recently many researchers like Devi [3], Gnanambal [4], Sheik John [16] have introduced and investigated several types of homeomorphisms in topological spaces. Navalagi and Mahesh Bhat [12] introduced the notion of sgp-closed set utilizing pre-closure operator. The notions of sgp-open sets, sgp-continuity are introduced in [12]. , recently Mahesh Bhat and mhd.hanif Page [ ]introduced and studied sgp-closed and sgp-open functions.

In this paper we continue the study of sgp-closed sets, with introducing and characterizing Strongly sgp-continuous and perfectly sgp continuous functions. We have also introduced completely sgp-continuous functions and studied some of its basic.

PRILIMINARYS: (x,τ) represents topological spaces on which no separation axioms are assumed unless explicitly stated. If A is any subset of space X, then Cl(A) and Int(A) denote the closure of A and the interior of A in X respectively. We use some definations :

Definition : A subset A of a topological space (X, τ) is called a

1. semi-open set [5] if $A \subseteq cl(int(A))$ and semi-closed set if $int(cl(A)) \subseteq A$.
2. pre-open set [10] if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.
3. α-open set [14] if $A \subseteq int(cl(int(A)))$ and α-closed set if $cl(int(cl(A))) \subseteq A$
4. semi pre open set [1] if $A \subseteq cl(int(cl(A)))$ and semi pre closed [1] if $int(cl(int(A))) \subseteq A$.
5. regular open set [9] if $A=int(cl(A))$ and a regular closed set if $A=cl(int(A))$.

<u>DEFINITION</u> 2.2. :

A subset A of a space X is said to be

i) generalized closed (g, closed)[6] if C1(A) whenever $A \subset U$ and U is open in X.

ii) generalized preclosed (gp – closed) [15] if pC1(A) Whenever $A \subset U$ and U is open.

iii) semi generalized pre closed (in short, sgp-closed) U and U is$\subseteq$ U, whenever A $\subseteq$set [12] if pCl(A) semi-open in X. The complement of sgp-closed set is sgp-open set and the family of all sgp-open sets of X is denoted by SGPO(X).

Properties of Strongly sgp-continuous and Perfectly sgp- continuous functions

Definition 3.1: A function f: X $\rightarrow$ Y is called strongly sgp-continuous if the inverse image of every sgp-open set in Y is open X.

Theorem 3.2: A function f: X $\rightarrow$ Y is strongly sgp-continuous if and only if the inverse image of every sgp -closed set in Y is closed in X.

Proof: Suppose f: X $\rightarrow$ Y is strongly sgp-continuous. Let G be a sgp-closed set in Y. Then Y – G is sgp-open set in Y. Since f is strongly sgp-continuous, f^{-1}(Y– G) is open in X. But f^{-1}(Y–G) = X –f^{-1}(G). Thus f^{-1}(G) is closed in X.

Conversely, suppose that the inverse image of every sgp-closed set in Y is closed in X. Let U be a sgp-open set in Y Then Y – U is sgp-closed in Y. By hypothesis, f^{-1}(Y – U) is closed set in X. But f^{-1}(Y – U) = X- f^{-1}(U) is closed in X. Therefore f^{-1}(U) is open in X. Hence f is strongly sgp –continuous function.

Theorem 3.3: Every strongly sgp-continuous function is a continuous function but not conversely.

Proof: Let f: X $\rightarrow$ Y be a strongly sgp-continuous function. Let G be an open set in Y. Then G is sgp-open set in Y. Since f is strongly sgp-continuous, f^{-1}(G) is open in X. Hence f is continuous function.

Example 3.4: Let X =Y={a, b, c}, τ = {X, ϕ, {a}, {a, c}} and σ = {Y, ϕ, {a}}. Let f: X $\rightarrow$ Y be the identity function. Then f is not sgp-continuous, since for the sgp-open set {a, b} in Y, f^{-1}({a, b}) = {a, b} is not open in X. However f is continuous function.

Theorem 3.5: Every strongly continuous function is strongly sgp-continuous.

Proof: Let f: X $\rightarrow$ Y be a strongly continuous function. Let G be a sgp-open set in Y. Then f^{-1}(G) is both open and closed in X. Therefore f^{-1}(G) is open in X. Hence f is strongly sgp-continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.6: Let X =Y={a, b, c}, τ = {X, ϕ, {a}, {a, b}} and σ = {Y, ϕ, {a, b}}. Then the identity function f: X $\rightarrow$ Y is strongly sgp-continuous but not strongly continuous, since for the subset {a, b} in Y, f^{-1}({a, b}) ={a, b} is open in X but not a closed set in X.

Theorem 3.7: If f: X $\rightarrow$ Y is continuous and Y is ${}_{sgp}T_c$-space. Then f is strongly sgp-continuous.

Proof: Let f: X $\rightarrow$ Y be a continuous function. Let G be a sgp-open set in Y. Then G is open in Y as Y is ${}_{sgp}T_c$-space. Since f is continuous, f^{-1}(G) is open in X. Hence f is strongly sgp-continuous function.

Theorem 3.8: If f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z is also strongly sgp-continuous functions, then their composition gof: X $\rightarrow$ Z is strongly sgp-continuous.

Proof: Let G be a sgp-open set in Z. Since g is strongly sgp-continuous, g^{-1}(G) is open in Y. Therefore g^{-1}(G) is sgp-open in Y. Again since f is strongly sgp-continuous, $f^{-1}(g^{-1}(G))$ is open in X. That is $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ which is open in X. Hence gof is strongly sgp-continuous.

Theorem 3.9: If f: X $\rightarrow$ Y is continuous and g: Y $\rightarrow$ Z is strongly sgp-continuous, then gof: X $\rightarrow$ Z is strongly sgp -continuous function.

Proof: Let G be a sgp-open set in Z. Since g is strongly sgp-continuous, g^{-1}(G) is open in Y. Then $f^{-1}(g^{-1}(G))$ is open in X as f is continuous. Therefore $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is open in X. Hence gof is strongly sgp-continuous function.

Theorem 3.10: If f: X $\rightarrow$ Y is sgp-continuous and g: Y $\rightarrow$ Z is strongly sgp-continuous, then their composition gof: X $\rightarrow$ Z is sgp-irresolute.

Proof: Let G be a sgp -open set in Z. Then g^{-1}(G) is open in Y as g is strongly sgp -continuous. Since f is sgp-continuous, $f^{-1}(g^{-1}(G))$ is sgp-open in X. But $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is sgp-open in X. Hence gof is sgp-irresolute.

Theorem 3.11: If f: X $\rightarrow$ Y is strongly sgp-continuous and g: Y $\rightarrow$ Z is continuous, then gof: X $\rightarrow$ Z is continuous function.

Proof: Let G be an open set in Z. Then g^{-1}(G) is open in Y since g is continuous. So g^{-1}(G) is sgp-open in Y. Again since f is strongly sgp-continuous, $f^{-1}(g^{-1}(G))$ is open in X. Therefore $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is open in X. Hence gof is continuous function.

4. Perfectly sgp-continuous function

Definition 4.1: A function f: X $\rightarrow$ Y is said to be perfectly sgp-continuous if the inverse image of every sgp-open set in Y is both open and closed in X.

Theorem 4.2: A function f: X $\rightarrow$ Y is perfectly sgp-continuous if and only if the inverse image of every sgp-closed set in Y is both open and closed in X.

Theorem 4.3: If a function f: X $\rightarrow$ Y is perfectly sgp-continuous, then f is strongly sgp-continuous function.

Proof: Let f: X $\rightarrow$ Y be a perfectly sgp-continuous function. Let G be a sgp-open set in Y. Then f^{-1}(G) is both open and closed in X. Therefore f^{-1}(G) is open in X. Hence f is strongly sgp-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 4.4: In Example 3.8, the function f is strongly sgp-continuous but not perfectly sgp-continuous, since for the sgp-open set {a, b} in Y, $f^{-1}(\{a, b\}) = \{a, b\}$ is open but not closed in X.

Theorem 4.5: Every perfectly sgp -continuous function is continuous function.

Proof: Let f: X $\rightarrow$ Y be a perfectly sgp-continuous function. Let G be an open set in Y. Then G is sgp-open in Y. Since f is perfectly sgp-continuous, $f^{-1}(G)$ is both open and closed in X. Therefore $f^{-1}(G)$ is open in X. Hence f is continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 4.6: In Example 3.4, the function f is continuous but not perfectly sgp -continuous, since for the sgp-open set {a, b} in Y, $f^{-1}(\{a, b\}) = \{a, b\}$ is not both open and closed in X.

Theorem 4.7: Every perfectly sgp-continuous function is perfectly continuous.

Proof: Let f: X $\rightarrow$ Y be a perfectly sgp-continuous function. Let G be an open set in Y. So G is sgp-open in Y. Since f is perfectly sgp continuous, $f^{-1}(G)$ is both open and closed in X. Hence f is perfectly continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 4.8: Let X =Y= {a, b, c}, $\tau = \{X, \phi, \{a\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}\}$. Let f: X $\rightarrow$ Y be the identity function. Then f is perfectly continuous but not perfectly sgp-continuous, since for the sgp-open set {a, b} in Y, $f^{-1}(\{a, b\}) = \{a, b\}$ is neither open nor closed in X.

Theorem 4.9: If a function f: X $\rightarrow$ Y is perfectly continuous and Y is ${}_{sgp}T_c$-space, then f is perfectly sgp-continuous.

Proof: Let G be a sgp-open set in Y. Then G is open in Y as Y is ${}_{sgp}T_c$-space. Since f is perfectly continuous, $f^{-1}(G)$ is both open and closed in X. Therefore f is perfectly sgp-continuous function.

Theorem 4.10: Let X be a discrete topological space, Y be any topological space and f: X $\rightarrow$ Y be a function. Then the following are equivalent.

i) f is perfectly sgp-continuous

ii) f is strongly sgp-continuous.

Proof: (i) $\Rightarrow$ (ii): Follows from the Theorem 4.3

(ii) $\Rightarrow$ (i): Let G be a sgp-open set in Y. By hypothesis, $f^{-1}(G)$ is open in X. Since X is discrete topological space, $f^{-1}(G)$ is closed in X. Therefore $f^{-1}(G)$ is both open and closed in X. Hence f is perfectly sgp-continuous function.

Theorem 4.11: If f: X $\rightarrow$ Y and g: Y $\rightarrow$ Z be two perfectly sgp-continuous functions, then gof: X $\rightarrow$ Z is perfectly sgp-continuous.

Proof: Let G be a sgp-open set in Z. Then $g^{-1}(G)$ is both open and closed in Y as g is perfectly sgp-continuous. So $g^{-1}(G)$ is sgp-open in Y. Since f is perfectly sgp-continuous, $f^{-1}(g^{-1}(G))$ is both open and closed in X. That is $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is both open and closed in X. Hence gof is perfectly sgp-continuous.

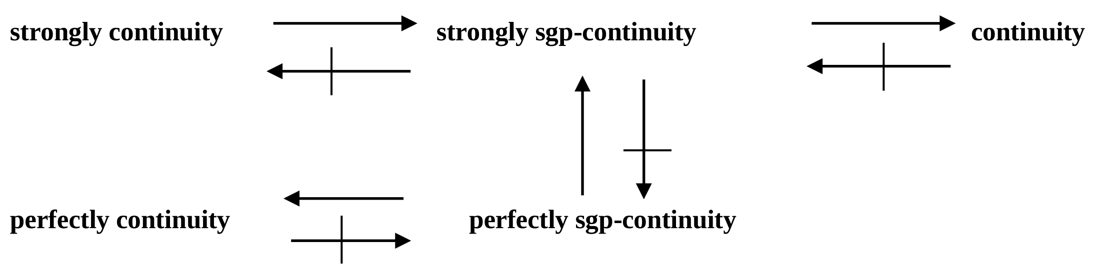
Theorem 4.12: If f: X $\rightarrow$ Y is perfectly sgp-continuous and g: Y $\rightarrow$ Z is sgp-irresolute, then gof: X $\rightarrow$ Z is perfectly sgp-continuous.

Proof: Let G be a sgp-open set in Z. Then $g^{-1}(G)$ is sgp-open in Y as g is sgp-irresolute. Since f is perfectly sgp-continuous, $f^{-1}(g^{-1}(G))$ is both open and closed in X. But $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is both open and closed in X. Hence gof is perfectly sgp-continuous.

Theorem 4.13: If f: X $\rightarrow$ Y is sgp-continuous and g: Y $\rightarrow$ Z strongly continuous then gof: X $\rightarrow$ Z is sgp-continuous.

Proof: Let G be an open set in Z. Then $g^{-1}(G)$ is both open and closed in Y as g is strongly continuous. So $g^{-1}(G)$ is open in Y. Since f is sgp-continuous, $f^{-1}(g^{-1}(G))$ is open in X. That is $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is open in X. Hence gof is sgp-continuous.

Remark 4.14: From the above results we have the following diagram.



Theorem 4.15: If f: X $\rightarrow$ Y is sgp-irresolute and g: Y $\rightarrow$ Z is perfectly sgp-continuous, then gof: X $\rightarrow$ Z is sgp-irresolute function.

Proof: Let G be a sgp-open set in Z. Since g is perfectly sgp-continuous, $g^{-1}(G)$ is both open and closed in Y. So $g^{-1}(G)$ is sgp-open in Y. Again since f is sgp-irresolute, $f^{-1}(g^{-1}(G))$ is sgp-open in X. That is $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is sgp-open in X. Hence gof is sgp-irresolute.

5. Completely sgp-continuous function

Definition 5.1: A function f: X → Y is said to be completely sgp-continuous if the inverse image of every sgp-open set in Y is regular-open in X.

Theorem 5.2: A function f: X → Y is completely sgp-continuous if and only if the inverse image of every sgp-closed set in Y is regular closed in X.

Proof: Suppose a function f: X → Y is completely sgp-continuous. Let F be a sgp-closed set in Y. Then Y- F is sgp-open in Y. Since f is completely sgp-continuous, f^{-1}(Y-F) is regular open in X. That is f^{-1}(Y –F) = X – f^{-1}(F) is regular open in X. Hence f^{-1}(F) is regular closed in X

Conversely, suppose that the inverse image of every sgp-closed set in Y is regular closed in X. Let K be a sgp-open set in Y. Then Y-K is sgp-closed in Y. By hypothesis, f^{-1}(Y-K) is regular closed in X. That isf^{-1}(Y-K) = X – f^{-1}(K) is regular closed set in X. Therefore f^{-1}(K) is regular open in X. Hence f is completely sgp-continuous function.

Theorem 5.3: If a function f: X → Y is completely sgp-continuous then f is continuous.

Proof: Let G be an open set in Y. Then G is sgp-open set in Y. Since f is completely sgp-continuous, f^{-1}(G) is regular open in X. So f^{-1}(G) is open in X. Hence f is continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 5.4: In Example 3.4, the function f is continuous but not completely sgp-continuous, since for the sgp-open set {a, b} of Y, f^{-1}({a, b}) = {a, b} is not regular open in X.

Theorem 5.5: Every completely sgp-continuous function completely continuous.

Proof: Let f: X → Y be a completely sgp-continuous function. Let G be an open set in Y. Then G is sgp-open in Y. Since f is completely sgp-continuous, f^{-1}(G) is regular-open in X. Hence f is completely continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 5.6: Let X =Y={a, b, c}, τ = {X, ϕ, {a}, {b}, {a, b}} and σ = {Y,ϕ, {a}}. Let f: X → Y be the identity function. Then f is completely continuous but not completely sgp-continuous, since for the sgp-open set {a, b} in Y, f^{-1}({a, b}) = {a, b} is not regular open in X.

Theorem 5.7: If a function f: X → Y is completely sgp-continuous then f is strongly sgp-continuous.

Proof: Let f: X → Y be completely sgp-continuous. Let G be sgp-open set in Y. Since f is completely sgp-continuous, f^{-1}(G) is regular open in X. Therefore f^{-1}(G) is open in X. Hence f is strongly sgp-continuous.

The converse of the above theorem need not be true as seen from the following example.

Example 5.8: In Example 3.3.74, the function f is strongly sgp-continuous but not completely sgp-continuous, since for the sgp-open set {a, b} in Y, f^{-1}({a, b}) = {a, b} is not regular open in X.

Theorem 5.9: If a function f: X → Y is completely continuous and Y is ${}_{sgp}T_c$-space, then f is completely sgp-continuous.

Proof: Let G be a sgp-open set in Y. Then G is an open in Y as Y is ${}_{sgp}T_c$-space. Since f is completely continuous, f^{-1}(G) is regular open in X. Therefore f is completely sgp-continuous function.

Theorem 5.10: If f: X → Y is completely continuous and g: Y → Z completely sgp-continuous then gof: X → Z is completely sgp-continuous.

Proof: Let G be a sgp-open set in Z. Then g^{-1}(G) is regular-open in Y as g is completely sgp-continuous. So g^{-1}(G) is open in Y. Since f is completely continuous, $f^{-1}(g^{-1}(G))$ is regular-open in X. That is $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is regular open in X. Hence gof is completely sgp-continuous.

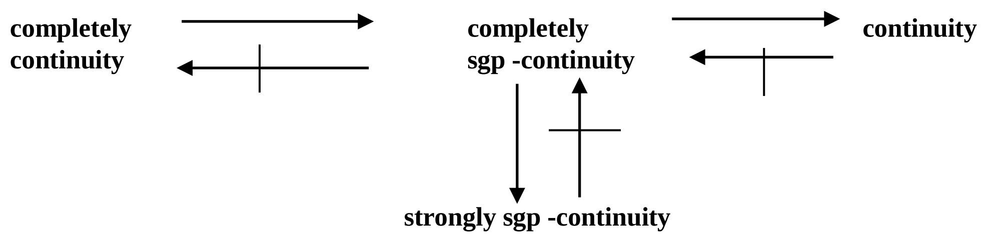
Theorem 5.11.: If f: X → Y is completely sgp-continuous and g: Y → Z is sgp-irresolute, then gof: X → Z is completely sgp-continuous.

Proof: Let G be a sgp-open set in Z. Since g is sgp-irresolute, g^{-1}(G) is sgp-open in Y. Since f is completely sgp-continuous, $f^{-1}(g^{-1}(G))$ is regular-open in X. That is $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is regular open in X. Hence gof is completely sgp-continuous.

Theorem 5.12: If f: X → Y is completely sgp-continuous and g: Y → Z strongly sgp-continuous, then gof: X → Z is completely sgp-continuous function.

Proof: Let G be a sgp-open set in Z. Since g is strongly sgp-continuous, g^{-1}(G) is open in Y. So g^{-1}(G) is sgp-open in Y. Again since f is completely sgp-continuous, $f^{-1}(g^{-1}(G))$ is regular-open in X. But $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is regular open in X. Hence gof is completely sgp-continuous.

Remark 5.13: From the above observations we get the following diagram.



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