IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

ON $\tau_1\tau_2$#RG-CONTINUOUS IN BITOPOLIGICAL SPACES AND $\tau_1\tau_2$#RG-IRRESOLUTE FUNCTIONS

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Abstract

In this paper we introduce $\tau_1\tau_2$#rg-closed sets and $\tau_1\tau_2$ #rg-open sets in bitopological spaces and established their relationships with some generalized sets in bitopological spaces. The aim of this paper is to introduce $\tau_1\tau_2$#rg-continuous functions and $\tau_1\tau_2$#rg-irresolute functions by using $\tau_1\tau_2$#rg-closed sets and characterize their basic properties.

Keywords: $\tau_1\tau_2$#rg-closed; $\tau_1\tau_2$#rg-open; $\tau_1\tau_2$#rg-continuous; $\tau_1\tau_2$#rg-irresolute.

1. Introduction

The concept of continuity is connected with the concept of topology. A weaker form of continuous functions called generalized continuous (briefly, g-continuous) maps was introduced and studied by Balachandran [1]. Then many researchers studied on generalizations of continuous maps. Recently, Sivanthi and Thilaga Leevathi [2] introduced and studied the properties of $\tau_1\tau_2$#rg-closed sets. The purpose of this paper is to introduce the concept of $\tau_1\tau_2$#rg-continuous and #rg-irresoluteness that are characterized and their relationship with weak and generalized continuity are investigated.

2. Preliminaries

Throughout this paper (X; τ_1, τ_2) and (Y, σ_1, σ_2) (or briefly, X and Y) represents a bitopological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a bitopological space X, $\tau_2$cl(A) and $\tau_1$int(A) denote the $\tau_2$closure of A and the $\tau_1$interior of A, respectively. X \ A or A^c denotes the complement of A in X. We recall the following definitions and results.

Definition 2.1 A subset A of a bitopological space (X, τ_1, τ_2) is called:

(1) $\tau_1\tau_2$ preopen set if A ⊆ $\tau_1$int$\tau_2$cl (A) and a $\tau_1\tau_2$preclosed set if $\tau_2$cl$\tau_1$int (A) ⊆ A.

(2) $\tau_1\tau_2$semiopen set[1] if A ⊆$\tau_2$cl$\tau_1$int (A) and a $\tau_1\tau_2$semiclosed set if $\tau_1$int$\tau_2$cl (A) ⊆A.

(3) $\tau_1\tau_2$ regular open set if A = $\tau_1$int$\tau_2$cl (A) and a $\tau_2$regular closed set if A = $\tau_2$cl$\tau_1$int (A).

(4) $\tau_1\tau_2\pi$- open set if A is a finite union of regular open sets.

(5) $\tau_1\tau_2$regular semi open if there is a $\tau_1$regular open U such U ⊆A ⊆$\tau_2$cl(U).

Definition:2.2 A subset A of (X,τ_1, τ_2) is called

(1) $\tau_1\tau_2$generalized closed set (briefly, $\tau_1\tau_2$g-closed) if $\tau_2$cl (A)⊆ U whenever A⊆U and U is open in X.

(2) $\tau_1\tau_2$regular generalized closed set (briefly, $\tau_1\tau_2$rg-closed) if $\tau_2$cl (A)⊆U whenever A⊆U and U is τ_1-regular open in X.

(3) $\tau_1\tau_2$generalized preregular closed set (briefly,$\tau_1\tau_2$gpr-closed) if $\tau_2$pcl (A)⊆U whenever A ⊆ U and U is τ_1-regular open in X.

(4) $\tau_1\tau_2$ regular weakely generalized closed set (briefly,$\tau_1\tau_2$wg-closed) if $\tau_2$cl$\tau_1$int (A) ⊆U whenever A⊆ U and U is τ_1- regular open in X.

(5)$\tau_1\tau_2$ rw-closed if $\tau_2$cl(A)⊆U whenever A⊆U and U is $\tau_1$regular semi open.
(6) $\tau_1\tau_2\#rg$-closed if $\tau_2$cl(A)⊆U whenever A⊆U and U is $\tau_1$rw-open.

The complements of the above mentioned closed sets are their respective open sets.

Definition 2.3 A map f : X→Y is called $\tau_1\tau_2$ g-continuous [1] (resp. $\tau_1\tau_2$ rg-continuous) if $f^{-1}(V)$ is g-closed (resp. $\tau_1\tau_2$ rg-closed) in X for every closed subset V of Y .

Definition 2.4 For a subset A of a space (X, τ_1,τ_2), $\tau_1\tau_2$#rg \ cl(A) = ⋂{F : A ⊆ F; F is $\tau_1\tau_2$#rg closed in X} is called the $\tau_1\tau_2$#rg-closure of A.

Definition 2.5 Let (X; τ_1 , τ_2) be a bitopological space and $\tau_{\tau_1\tau_2\#rg}$ = {V ⊆ X : $\tau_1\tau_2$#rg - $\tau_2$cl(X \ V) = X \ V}
.

Lemma 2.6 For any x ∈ X, x ∈ $\tau_1\tau_2$#rg-τ_2 cl(A) if and only if V∩ A ≠ ∅ for every $\tau_1\tau_2$#rg-open set V containing x.

Lemma 2.7 Let A and B be subsets of (X; τ_1,τ_2). Then:
(1) #rg -$\tau_2$cl(∅) = ∅ and $\tau_1\tau_2$#rg - $\tau_2$cl(X) = X.
(2) If A ⊆ B, then $\tau_1\tau_2$#rg - $\tau_2$cl(A) ⊆ $\tau_1\tau_2$#rg -τ_2 cl(B).
(3) A ⊆$\tau_1\tau_2$#rg -τ_2 cl(A).
(4) If A is $\tau_1\tau_2$#rg-closed, then $\tau_1\tau_2$#rg -$\tau_2$cl(A) = A.
(5) $\tau_1\tau_2$#rg-closure of a set A is not always $\tau_1\tau_2$#rg-closed.

Remark 2.8 Suppose $\tau_{\tau_1\tau_2\#rg}$ is a bitopology. If A is $\tau_1\tau_2$#rg-closed in (X; τ_1,τ_2), then A is closed in (X,$\tau_{\tau_1\tau_2\#rg}$).

Lemma 2.9 A set A ⊆ X is $\tau_1\tau_2$#rg-open if and only if F ⊆ $\tau_1$intA whenever F ⊆ A, F is $\tau_1\tau_2$rw-closed.

3. $\tau_1\tau_2$#RG-Continuous Functions

In this section, we introduce and study $\tau_1\tau_2$#rg-continuous functions.

Definition 3.1 A function f : X→Y is called #rg-continuous if f^{-1} (V) is $\tau_1\tau_2$#rg-closed in (X, τ_1,τ_2) for every closed subset V of (Y, σ_1, σ_2).

Theorem 3.2 Every continuous map is $\tau_1\tau_2$#rg-continuous map
Proof Let f : X→Y is continuous map then for every closed set A in Y , f^{-1} (A) is closed in X. Since every closed set is $\tau_1\tau_2$#rg-closed, f^{-1} (A) is $\tau_1\tau_2$#rg-closed in X. Hence f is $\tau_1\tau_2$#rg-continuous map.

Example 3.3 Let X ={a, b, c} with topologies τ_1 ={∅, {a}, {b},{a, b}, X} and τ_2 ={∅, {a}, {a, b},X}. Let Y = {1, 2, 3, 4, 5} with topologies σ_1 ={∅, {1}, {2, 3, 4, 5}, Y} and σ_2 ={∅, {2}, {3}, {2, 3}, {2, 3, 4, 5},Y}. A function F: (X, τ_1, τ_2)⟶(Y, σ_1, σ_2) is defined as follows: $F(a)$ ={2, 3}, $F(b)$ ={1, 2}, $F(c)$ = {1, 4, 5}. Then, F is $\tau_1\tau_2$#rg-continuous.

Corollary 3.1 Every $\tau_1\tau_2$regular continuous map is $\tau_1\tau_2$#rg-continuous
Proof Follows from Theorem 3.2 and the fact that every $\tau_1\tau_2$regular continuous map is $\tau_1\tau_2$continuous.

Theorem 3.4 Every $\tau_1\tau_2$#rg-continuous map is $\tau_1\tau_2$g-continuous map (resp. $\tau_1\tau_2$ rg-continuous).
Proof Suppose f : X→Y is $\tau_1\tau_2$#rg-continuous. Let V be a closed set in Y. Since f is $\tau_1\tau_2$#rg-continuous, then f^{-1} (V) is $\tau_1\tau_2$#rg-closed set in X. Since every $\tau_1\tau_2$#rg-closed set is $\tau_1\tau_2$g-closed (resp. $\tau_1\tau_2$rg-closed) set, then f^{-1} (V) is also$\tau_1\tau_2$ g-closed (resp. $\tau_1\tau_2$rg-closed) set in X. Thus f is $\tau_1\tau_2$g-continuous (resp. $\tau_1\tau_2$ rg-continuous).

The converse of the above theorem is not necessarily true as seen from the following example.

Example 3.5 Let X = Y ={a, b, c}with topologies τ_1 ={∅, {a}, X}, τ_2 ={∅, {a}, {a, b},X}, σ_1={∅, {b, c}, Y} and σ_2={∅, {b,c}, {c},Y}, define f : X→Y by f(a) = b, f(b) = a and f(c) = c then f is $\tau_1\tau_2$g-continuous but not $\tau_1\tau_2$#rg-continuous.

Corollary 3.2 Every $\tau_1\tau_2$ #rg-continuous is $\tau_1\tau_2$ rwg-continuous and $\tau_1\tau_2$gpr-continuous.

Proof Follows from Theorem 3.4 and the fact that every $\tau_1\tau_2$rg-continuous map is $\tau_1\tau_2$rwg-continuous and $\tau_1\tau_2$gpr-continuous.

Corollary 3.3 Every $\tau_1\tau_2$#rg-continuous is $\tau_1\tau_2$gs-continuous.
Proof Follows from Theorem 3.4 and the fact that every g-continuous map is gs-continuous.

Corollary 3.4 Every $\tau_1\tau_2$#rg-continuous is $\tau_1\tau_2$gsp-continuous.
Proof Follows from Corollary 3.3 and the fact that every $\tau_1\tau_2$gs-continuous map is $\tau_1\tau_2$gsp-continuous.

Theorem 3.6 Let f : X→Y be a function. Then the following are equivalent:
(1) f is $\tau_1\tau_2$#rg-continuous,
(2) The inverse image of each open set in Y is $\tau_1\tau_2$#rg-open in X.
(3) The inverse image of each closed set in Y is $\tau_1\tau_2$#rg-closed in X.
Proof
Suppose (1) holds. Let G be open in Y . Then Y \ G is closed in Y . By (1) f^{-1} (Y \ G) is $\tau_1\tau_2$#rg-closed in X. But f^{-1} (Y \ G) = X \ f^{-1} (G) which is $\tau_1\tau_2$#rg-closed in X. Therefore f^{-1} (G) is $\tau_1\tau_2$#rg-open in X. This proves (1) ⇒ (2).

Suppose (2) holds. Let V be any closed set in Y. Then Y \ V is open set in Y . By (2), f^{-1} (Y \ V) is $\tau_1\tau_2$#rg-open. But f^{-1} (Y \ V) = X \ f^{-1} (V) which is $\tau_1\tau_2$#rg-open. Therefore f^{-1} (V) is $\tau_1\tau_2$#rg-closed. This proves (2) ⇒ (3).

The implication (3) ⇒ (1) follows from Definition 3.1.

Theorem 3.7 If a function f : X→Y is $\tau_1\tau_2$#rg-continuous, then f($\tau_1\tau_2$#rg - τ_2 cl(A)) ⊆ τ_2 cl(f(A)) for every subset A of X.

Proof
Let f : X→Y be $\tau_1\tau_2$#rg-continuous. Let A ⊆ X. Then $\tau_2$cl(f(A)) is closed in Y . Since f is $\tau_1\tau_2$#rg-continuous, f^{-1} ($\tau_2$cl(f(A)) is $\tau_1\tau_2$#rg-closed in X and A ⊆ f^{-1} (f(A)) ⊆ f^{-1} ($\tau_2$cl(f(A)), implies $\tau_1\tau_2$#rg \ $\tau_2$cl(A) ⊆ f^{-1} ($\tau_2$cl(f(A)). Hence f($\tau_1\tau_2$#rg - $\tau_2$cl(A)) ⊆ $\tau_2$cl(f(A)).

Theorem 3.8 Let X be a space in which every singleton set is $\tau_1\tau_2$rw-closed. Then f : X→Y is $\tau_1\tau_2$#rg-continuous , if $x \in \tau_1$int (f^{-1} (V)) for every open subset V of Y contains f(x).

Proof Suppose f : X→Y is $\tau_1\tau_2$#rg-continuous. Fix x ∈ X and an open set V in X such that f(x) ∈ V . Then f^{-1} (V) is $\tau_1\tau_2$#rg-open. Since x ∈ (f^{-1} (V) and {x} is $\tau_1\tau_2$rw-closed, x∈ $\tau_1$int(f(f^{-1} (V)) by Lemma 2.9.

Conversely, assume that x ∈ $\tau_1$int((f^{-1} (V)) for every open subset V of Y containing f(x). Let V be an open set in Y. Suppose F ⊆ f^{-1} (V) and F is $\tau_1\tau_2$rw-closed. Let x ∈ F, then f(x) ∈V so that x ∈ $\tau_1$int((f^{-1} (V)). That implies F ⊆ $x \in \tau_1$int((f^{-1} (V)). Therefore by Lemma 2.9, f^{-1} (V) is $\tau_1\tau_2$#rg-open. This proves that f is #rg-continuous.

Theorem 3.9 Let f : X→Y be a function. Let X and Y be any two spaces such that $\tau_{\tau_1\tau_2\#rg}$ is a bitopology on X. Then the following statements are equivalent:
(1) For every subset A of X, f($\tau_1\tau_2$#rg \ $\tau_2$cl(A)) ⊆ $\tau_2$cl(f(A)) holds,
(2) f : (X; $\tau_{\tau_1\tau_2\#rg}$) → (Y, σ_1, σ_2) is continuous.
Proof Suppose (1) holds. Let A be closed in Y . By hypothesis f($\tau_1\tau_2$#rg \ $\tau_2$cl(f^{-1} (A))) ⊆ $\tau_2$cl(f((f^{-1} (A))) ⊆ $\tau_2$cl(A) = A. i.e., $\tau_1\tau_2$#rg - τ_2 cl(f^{-1} (A)) ⊆ f^{-1} (A). Also f^{-1} (A) ⊆ $\tau_1\tau_2$#rg - τ_2 cl((f^{-1} (A)). Hence, $\tau_1\tau_2$#rg - $\tau_2$cl((f^{-1} (A)) = f^{-1} (A). This implies ((f^{-1} (A))^c ∈ $\tau_{\tau_1\tau_2\#rg}$ Thus f^{-1} (A) is closed in (X; $\tau_{\tau_1\tau_2\#rg}$) and so f is continuous. This proves (2).

Suppose (2) holds. For every subset A of X, $\tau_2$cl(f(A)) is closed in Y . Since f : (X, $\tau_{\tau_1\tau_2\#rg}$) → (Y, σ_1, σ_2) is continuous, (f^{-1} ($\tau_2$cl(f(A))) is closed in (X, $\tau_{\tau_1\tau_2\#rg}$)) that implies by Definition 2.5 $\tau_1\tau_2$#rg - $\tau_2$cl((f^{-1} ($\tau_2$cl(f(A))) = (f^{-1} ($\tau_2$cl(f(A)). Now we have, A ⊆ (f^{-1} (f(A)) ⊆ (f^{-1} ($\tau_2$cl(f(A)) and by Lemma 2.7 (2), $\tau_1\tau_2$rg - $\tau_2$cl(A) ⊆$\tau_1\tau_2$ #rg ⊆ $\tau_2$cl((f^{-1} ($\tau_2$cl(f(A)) = (f^{-1} ($\tau_2$cl(f(A)). Therefore f($\tau_1\tau_2$#rg - τ_2 cl(A)) ⊆ $\tau_2$cl(f(A)).

Theorem 3.10 Let X, Y and Z be bitopological spaces such that $\sigma_{\tau_1\tau_2\#rg} = \sigma$. Let f : X→Y and g : Y→ Z be $\tau_1\tau_2$#rg-continuous functions. Then the composition g o f : X→Z is $\tau_1\tau_2$ #rg-continuous.

Proof Let V be closed in (Z, μ_1, μ_2). Since g is $\tau_1\tau_2$#rg-continuous, g^{-1}(V) is $\tau_1\tau_2$#rg-closed in Y. Since $\sigma_{\tau_1\tau_2\#rg} = \sigma$, by Remark 2.8. g^{-1}(V) is closed in Y. Since f is $\tau_1\tau_2$#rg-continuous, (f^{-1} (g^{-1}(V)) is $\tau_1\tau_2$#rg-closed.
i.e. (g of)$^{-1}$(V) is $\tau_1\tau_2$#rg-closed in X. Therefore g o f is $\tau_1\tau_2$#rg-continuous.

4. $\tau_1\tau_2$#RG-Irresolute Functions

In this section $\tau_1\tau_2$#rg-irresolute function is introduced and their basic properties are discussed.

Definition 4.1 A function f : X→Y is called $\tau_1\tau_2$#rg-irresolute if f^{-1} (V) is $\tau_1\tau_2$#rg-closed in X for every $\tau_1\tau_2$#rg-closed subset V of Y.

Theorem 4.2 Every $\tau_1\tau_2$#rg-irresolute function is $\tau_1\tau_2$#rg-continuous but converse is not necessarily true.

Proof Suppose f : X→Y is $\tau_1\tau_2$#rg-irresolute. Let V be any closed subset of Y, then V is $\tau_1\tau_2$#rg- closed set in Y . Since f is $\tau_1\tau_2$#rg-irresolute, f^{-1} (V) is $\tau_1\tau_2$#rg-closed in X. Hence f is $\tau_1\tau_2$#rg-continuous.

The converse of the above theorem need not be true as seen from the following example.
Example 4.3 Let X = Y = {a, b, c, d}with topologies τ_1 ={∅, {a}, {b},{a, b}, {a, b, c}, X} and τ_2 ={∅, {a}, {a, b},X}, σ_1 ={∅, ={∅, {a}, {b},{a, b}, {a, b, c},Y} and σ_2 ={∅, , {a}, {b},{a, b}, Y}. Define f : X→Y by identity mapping then f is $\tau_1\tau_2$ #rg-continuous but not $\tau_1\tau_2$#rg-irresolute.

Theorem 4.4 If map f : X→Y is $\tau_1\tau_2$#rg-continuous map and Y is $T_{\tau_1\tau_2\#rg}$ -space, then f is $\tau_1\tau_2$#rg-irresolute.

Proof Let f : X→Y is $\tau_1\tau_2$#rg-continuous map then inverse image of every closed set in Y is $\tau_1\tau_2$#rg-closed set in X. Since Y is T#rg-space, inverse image of every $\tau_1\tau_2$#rg-closed set in Y is $\tau_1\tau_2$#rg-closed set in X. i.e., f is $\tau_1\tau_2$#rg-irresolute.

Theorem 4.5 Let f : X→Y be $\tau_1\tau_2$rw-irresolute and closed. Then f maps a $\tau_1\tau_2$#rg-closed set in X into a $\tau_1\tau_2$#rg-closed set in Y.
Proof Let A be $\tau_1\tau_2$#rg-closed in X. Let f(A) ⊆ U, where U is $\tau_1\tau_2$rw-open. Then A ⊆ f^{-1} (U). Since f is $\tau_1\tau_2$rw-irresolute, f^{-1} (U) is $\tau_1\tau_2$rw-open in X. Since A is $\tau_1\tau_2$#rg-closed, $\tau_2$cl(A) ⊆ f^{-1} (U) that implies f($\tau_2$cl(A)) ⊆U

Since f is closed f($\tau_2$cl(A)) is closed that implies $\tau_2$cl(f(A)) ⊆ $\tau_2$cl(f($\tau_2$clA)) = f($\tau_2$cl(A)) ⊆ U.
Hence f(A) is $\tau_1\tau_2$#rg-closed in Y.

Theorem 4.6 Let f : X→Y and g : Y → Z be any two functions. Let h = gof. Then:
(1) h is $\tau_1\tau_2$#rg-continuous if f is $\tau_1\tau_2$#rg-irresolute and g is $\tau_1\tau_2$#rg-continuous,
(2) h is $\tau_1\tau_2$#rg-irresolute if both f and g are both $\tau_1\tau_2$ #rg-irresolute and,
(3) h is $\tau_1\tau_2$#rg-continuous if g is continuous and f is $\tau_1\tau_2$#rg-continuous.
Proof Let V be closed in Z. (1) Suppose f is $\tau_1\tau_2$#rg-irresolute and g is $\tau_1\tau_2$#rg-continuous. Since g is $\tau_1\tau_2$#rg-continuous, g^{-1}(V) is $\tau_1\tau_2$#rg-closed in Y . Since f is $\tau_1\tau_2$#rg-irresolute, using Definition 4.1, f^{-1} (g^{-1}(V)) is $\tau_1\tau_2$#rg-closed in X. This proves (1).

(2) Let f and g be both $\tau_1\tau_2$#rg-irresolute. Then g^{-1}(V) is $\tau_1\tau_2$#rg-closed in Y . Since f is $\tau_1\tau_2$#rg-irresolute, using Definition 4.1 f^{-1} (g^{-1}(V)) is #rg-closed in X. This proves (2).

(3) Let g be continuous and f be $\tau_1\tau_2$#rg-continuous. Then g^{-1} (V) is closed in Y . Since f is $\tau_1\tau_2$#rg-continuous, using Definition 3.1, f^{-1} (g^{-1} (V)) is $\tau_1\tau_2$#rg-closed in X. This proves (3).

The next theorem follows easily as a direct consequence of definitions.

Theorem 4.7 A function f : X→Y is $\tau_1\tau_2$#rg-irresolute if and only if the inverse image of every $\tau_1\tau_2$#rg-open set in Y is $\tau_1\tau_2$#rg-open in X.

Definition 4.8 A function f : X→Y is said to be $\tau_1\tau_2$#rg-closed (resp. $\tau_1\tau_2$#rg-open) if for every #rg-closed (resp. $\tau_1\tau_2$#rg-open) set U of X the set f(U) is $\tau_1\tau_2$#rg-closed (resp. $\tau_1\tau_2$#rg-open) in Y .

Theorem 4.9 Let f : X→Y be a bijection. Then the following are equivalent:
(1) f is $\tau_1\tau_2$#rg-open,
(2) f is $\tau_1\tau_2$#rg-closed,
(3) f^{-1} is $\tau_1\tau_2$#rg-irresolute.
Proof Suppose f is $\tau_1\tau_2$#rg-open. Let F be $\tau_1\tau_2$#rg-closed in X. Then X \ F is $\tau_1\tau_2$#rg-open. By Definition 4.8.
f(X \ F) is $\tau_1\tau_2$#rg-open. Since f is a bijection, Y \ f(F) is $\tau_1\tau_2$#rg-open in Y . Therefore f is $\tau_1\tau_2$#rg-closed.
This proves (1) ⇒ (2).
Let g = f^{-1}. Suppose f is $\tau_1\tau_2$#rg-closed. Let V be $\tau_1\tau_2$#rg-open in X. Then X \ V is $\tau_1\tau_2$#rg-closed in X.
Since f is $\tau_1\tau_2$#rg-closed, f(X \ V) is $\tau_1\tau_2$#rg-closed. Since f is a bijection, Y \ f(V) is $\tau_1\tau_2$#rg-closed that implies f(V) is $\tau_1\tau_2$#rg-open in Y . Since g = f^{-1} and since g and f are bijection g^{-1}(V) = f(V) so that g^{-1}(V) is $\tau_1\tau_2$#rg-open in Y . Therefore f^{-1} is $\tau_1\tau_2$#rg-irresolute. This proves (2) ⇒ (3).

Suppose f^{-1} is $\tau_1\tau_2$#rg-irresolute. Let V be $\tau_1\tau_2$#rg-open in X. Then X \ V is $\tau_1\tau_2$#rg-closed in X. Since f^{-1} is $\tau_1\tau_2$#rg-irresolute and $(f^{-1})^{-1}$(X \ V) = f(X \ V) = Y \ f(V) is $\tau_1\tau_2$#rg-closed in Y that implies f(V) is $\tau_1\tau_2$#rg-open in Y . Therefore f is $\tau_1\tau_2$ #rg-open. This proves (3) ⇒ (1).

Theorem 4.10 Let f : X→Y and g : Y → Z be two functions. Suppose f and g are $\tau_1\tau_2$#rg-closed (resp. $\tau_1\tau_2$#rg-open). Then g o f is $\tau_1\tau_2$#rg-closed (resp. $\tau_1\tau_2$#rg-open).

Proof Let U be any $\tau_1\tau_2$#rg-closed (resp. $\tau_1\tau_2$#rg-open) set in X. Since f is $\tau_1\tau_2$#rg-closed, using Definition 4.8, f(U) is $\tau_1\tau_2$#rg-closed (resp. $\tau_1\tau_2$#rg-open) in Y . Again since g is $\tau_1\tau_2$#rg-closed (resp. $\tau_1\tau_2$#rg-open), using Definition 4.8, g(f(U)) is $\tau_1\tau_2$#rg-closed (resp. $\tau_1\tau_2$#rg-open) in Z. This shows that gof is $\tau_1\tau_2$#rg-closed (resp. $\tau_1\tau_2$#rg-open).

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