VALUE JUDGMENT: AN ENQUIRY INTO THEORETICAL WELFARE ECONOMICS

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ABSTRACT

The welfare economists have been confronted with the controversies of interpersonal comparisons or of value judgments for a long period of time. Following Pareto most of the conventional theory of welfare economics rested on the assumed value judgment that if one person was better off and no one was worse off welfare was increased. But without the knowledge of utility or welfare function none can be sure that satisfying those conditions is better than violating them. Moreover Paretian value judgment did not apply to a situation where some persons were benefited and some were harmed by some policy change. . Professor Amartya Kumar Sen in his article "Interpersonal Aggregation and Partial Comparability", Econometrica 38, May1970, has made an attempt to provide a fairly rigorous presentation of a possible framework of interpersonal comparability. In this paper I have found out how far Prof. Sen's partial comparability analysis suits our practical problem of evaluation of alternative social states in respect of social welfare. At the same time I have tried to point out unexplored part of the problems of measurement of social welfare and comparability. In course of my exploration I have kept it in my mind that both welfare and non-welfare information constitute the appropriate basis of social welfare evaluation**.**

Key Words: Value Judgment, Welfare, Interpersonal Comparability

Taking the objective of a society to be the maximization of welfare of all of its members economists ran into an intractable problem of measurability both at individual level and at the level of society. The welfare economists then confronted with the controversies of interpersonal comparisons or of value judgments. However the popular belief that interpersonal comparison of well-being requires measurable individual well-being had been side tracked with the development of the New Welfare Economics. The founder of New Welfare Economics was Vilfredo Pareto. He not only used the concept of ordinal preferences but also defined optimum position, which was independent of any necessity of adding satisfactions or comparing satisfactions of different individuals. Pareto defined an optimum position to be one in which it was impossible to put any individual on a higher indifference curve or on a higher behaviour line without causing someone to drop to a lower one. Following Pareto most of the conventional theory of welfare economics rested on the assumed value judgment that if one person was better off and no one was worse off welfare was increased. The famous Paretian condition is necessary but not sufficient as satisfaction of the conditions of efficiency in production and exchange is necessary as violation of any one of them would make it possible to make some persons better off without making any one worse off. But fulfillment of these conditions are not sufficient for the achievement of Paretian optimum as without the knowledge of utility or welfare function none can be sure that satisfying those conditions is better than violating them. Moreover Paretian value judgment did not apply to a situation where some persons were benefited and some were harmed by some policy change. Welfare economics is not so much concerned with changes in the welfare of individuals as such. It requires a criterion of an increase in the welfare of individuals because the welfare of the community is regarded as a logical construction from the welfares of individuals. The possibility of extending the analysis to encompass such non-paretian changes has been the theme of the 'compensation principle'. The concept underlying the compensation principle is that if a change in policy would result in some persons being better off and some worse off and the gainers could compensate the losers in such a way that on balance every body was better off then welfare would be increased by implementing that change. Considerable debate has resulted on the issue of whether it is sufficient that adequate compensation could be made or whether it is necessary for the inference that compensation actually be made. Regarding the efficacy of such compensation criterion I.M.D.Little was of the opinion that any one would scarcely want to say that all

changes such that gainers could overcompensate the losers, must be good for it would all depend on who the uncompensated losers were.

Coming to the case of interpersonal comparability we note that two polar cases of interpersonal comparability seem to have received all the attention in the literature so far. Either it is assumed that individual welfare measures are fully comparable (Marshall) or that they are not comparable at all (Robbins). It is clear however that we frequently make judgments that are not consistent with non-comparability but which do not require full comparability. There emerge the cases intermediate between non-comparability and full comparability.

Judgments about social welfare are intimately connected with possibilities of interpersonal comparability of individual welfare. The type of interpersonal comparability needed for various types of judgments varies a great deal. For example, in comparing the sums of individual welfare levels for distinct alternatives, as under utilitarianism, what we take as origins of the respective individual welfare functions of different persons makes no difference to the ordering of the alternatives, because the origins get subtracted out in pair wise comparison. Origins thus need not be comparable for rankings of aggregate welfare, but comparability of units of individual welfare is obviously required. In contrast if we take some criteria of justice such as that of J.Rawls where the social ordering is based on comparing the welfare levels of the worst off individuals, origins are clearly important. On the other hand, we do not need at all a cardinal measure of individual welfare levels for the Rawls ordering, thus comparability of welfare units is irrelevant and all we need compare are absolute levels of welfare.

The argument, put forward by P.Diamond in his article " Cardinal Welfare, Individualistic Ethics and Interpersonal Comparisons of Utility" JPE 75, Oct 1976, depends crucially on the individual welfare levels and thus also origins being comparable. But the argument put forward by J.Harsanyi in his article " Cardinal Welfare, Individualistic Ethics and Interpersonal Comparisons of Utility", *JPE* 63, Aug 1955, requires only levels of welfare to be comparable as his model is concerned only with aggregate welfare. The basic problem that lies with both Diamond and Harsanyi is that neither of them states explicitly what precise assumptions they have made. Professor Amartya Kumar Sen in his article "Interpersonal Aggregation and Partial Comparability", Econometrica 38, May1970, has made an attempt to provide a fairly rigorous presentation of a possible framework of interpersonal comparability.

Let x be the set of alternative social states, x. Every individual i has a set L_i of real valued welfare functions, W_i, each defined over X. If individual welfare is 'ordinally measurable', then every element of L_i is a positive monotonic transformation of every other element, and furthermore every positive monotonic transformation of any element of L_i belongs to L_i. If on the other hand, individual welfare is 'cardinally measurable', then every element of L_i is a positive linear transformation of every other element, and every positive linear transformation of any element of L_i belongs to L_i. Now any element of the Cartesian product $L = \prod_{i=1}^{n} L_i$ constitutes n-tuple of individual welfare functions and is called

Functional Combination. At the same time L specifies all the possible n-tuples of individual welfare

functions. Depending on the types of measurability-comparability assumptions we can specify the comparison set $\bar{L}$ consisting of the set of admissible n tuples such that $\bar{L} \subset L$ and we declare that x has at least as much aggregate welfare as y, for any pair x, y, if and only if the sum of the individual welfare differences between x and y is non-negative for every element W of $\bar{L}$, i.e., $\forall x, y \in X : [xR^a y \leftrightarrow \forall W \in \bar{L} : \sum_i [W_i(x) - W_i(y)] \geq 0]$.. Here $\bar{L}$ depends on the measurability-comparability assumption as well as on the actual welfare situation. If for any welfare n-tuple $\{W_i^*\}$ belonging to $\bar{L}$, $\bar{L}$ consisting of exactly all welfare n-tuples $\{W_i\}$ such that:

(1) there exists some positive affine transformation ψ for which $W_i = \psi(W_i^*)$, for all i, we have the case of Cardinal Full Comparability (CFC).

(2) there exists some positive monotonic transformation ψ for which $W_i = \psi(W_i^*)$, for all i, we have the case of Ordinal Level Comparability (OLC).

(3) there exists a positive real number b and an $n-vector$ a for which $W_i = a_i + bW_i^*$, for all i, we have the case of Cardinal Unit Comparability (CUC).

(4) there exists an n-tuple of positive affine transformations $\{\psi_i\}$ for which $W_i = \psi_i(W_i^*)$, for all i, we have the case of Cardinal Non Comparability (CNC).

(5) there exists an n-tuple of positive monotonic transformations $\{\psi_i\}$ for which $W_i = \psi_i(W_i^*)$, for all i, we have the case of Ordinal Non Comparability (ONC).

Infact alternative approaches to social welfare evaluation can be subjected to informational analysis examining each approach in terms of the types of information that it admits and the types it excludes. The analysis begins with the general class of SWFLs where in line with a SWF Social Welfare Functional (SWFL) is defined to be a functional relation that specifies one and only one social ordering R over X, for any W i.e., for any $n-tuple$ of individual welfare functions, $w_1, w_2 .... w_n$, each defined over X. The specification of a SWFL is supplemented by an *invariance* requirement over the set of $n-tuples$ that reflect the same welfare situation given the measurability and comparability assumptions. Of the distinguished cases of measurability-comparability frameworks characterized above, the *most* demanding informational set-up, implying the *least* demanding invariance requirement is given by Cardinal Full Comparability (CFC). The *least* demanding informational set-up, implying the *most* demanding invariance requirement is given by Ordinal Non Comparability (ONC). If we know nothing about the social states and persons involved, unrestricted domain is a sensible assumption, but not necessarily so if we do know something and wish to use that information.

If we analyze Arrow's 'impossibility theorem' showing the impossibility of a Social Welfare Function mapping the set of $n-tuples$ of individual orderings to the set of social ordering satisfying the conditions of unrestricted domain (U), Pareto principle (P), independence of irrelevant alternatives (I), non-dictatorship (D), we note that cardinality is out and no interpersonal comparisons are brought in. Moreover individual orderings are based on actual preferences of people. Here the extremely narrow informational base of collective choice is held responsible for the persistence of the problem of forming

judgments on the basis of individuals' actual preferences and welfares without any interpersonal comparisons and/or cardinality.

The picture does not change substantially if we base social welfare judgments on individual cardinal welfares without interpersonal comparability. Whatever we gain by introducing cardinality of individual welfare functions we fail to make any use of it due to non-comparability. The inability to say anything on the relative well-being of different persons and on their relative gains and losses makes this approach unsuitable for welfare judgments.

If interpersonal comparability is introduced without cardinality, it is possible to base social welfare judgments on relative levels of welfare of different persons. Using such comparability Suppes (1966) has proposed a partial ordering which uses the notion of dominance more widely than the Pareto principle. But in this framework welfare differences can not be compared given the ordinal nature of individual welfare functions and interpersonal comparisons.

There is, however, at least one criterion, viz, Rawls' (1971) 'maximin' conception of justice, which deliberately avoids comparisons of gains and losses. The Rawls criterion requires the maximization of the welfare level of the worst-off person and x is preferred to y if and only if the worst-off person in x is better off than the worst-off person in y. Symbolically:

xRy if and only if $\exists k : [\forall i : (x,i)\tilde{R}(y,k)]$. This concentration on the level of welfare of only one person makes the criterion rather an extremist one. The extremism of the criterion has attracted a lot of attention because of its concentration on the level of welfare of only the worst-off person. But the most interesting aspect of Rawls' departure from earlier approaches lies in the fact of basing social preference on the levels of individual welfare without regard to cardinal measures that permit comparisons of gains and losses. The idea of giving priority to the interests of a person who is going to be worse-off any way compared with another was captured much more generally in an equity axiom suggested by Hammond. Hammond's Equity Axiom states that if for any pair of social states $x, y,$ for some personal welfare n-tuple $\{W_i\},$ it is the case that for two persons g and h: $W_g(y) < W_g(x) < W_h(x) < W_h(y),$ and for all $i \neq g, h : W_i(x) = W_i(y),$ then $xRy.$ The use of Hammond's Equity Axiom tends to convert the informational framework of cardinal full comparability effectively into one of ordinal level comparability since Hammond's Equity Axiom is based on comparisons of levels with no attention being paid to the magnitudes of the gains and losses of the persons involved. Comparisons of 'units' play a crucial role in calculating 'net advantages' in an aggregative framework and this is the focus of utilitarianism. On the other hand the notion of 'equity' involves special consideration being given to the badly off and this does bring in comparisons of welfare levels.

The utilitarian approach requires cardinality and comparability of units but not levels. The main deficiency of unit comparability lies in its extreme difficulty of providing a rationale for assuming welfare units to be comparable without welfare levels being so. It is due to the fact that adding a constant to one person's welfare function without doing the same for the others can change the relative

levels of welfare substantially, but does not affect the utilitarian ranking since $[W_i(x) - W_i(y)]$, for each i, remain unaffected.

Cardinal Full Comparability requires cardinality and comparability of both units and levels.

Now it is to be noted that a SWF is a special case of SWFL, in which only the individual ordering properties are used. It may also be remarked that the aggregation relation for any $W \in L$ is a SWFL. Now Corresponding to Arrow's conditions on a SWF, similar conditions are imposed on a SWFL.

CONDITION $\bar{U}$: (*Unrestricted Domain*): The domain of the SWFL includes all logically possible W, for example, all possible n-tuples of individual welfare functions defined over X.

CONDITION $\bar{I}$: (*Independence of irrelevant alternatives*): If for all i, $W_i(x) = \hat{W}_i(x)$ and $W_i(y) = \hat{W}_i(y)$, for some pair $x, y \in X$, for some pair of welfare combinations W and $\hat{W}$, then $xRy \leftrightarrow x\hat{R}y$ where R and $\hat{R}$ are the social orderings corresponding to W and $\hat{W}$.

CONDITION $\bar{D}$: (*Non-Dictatorship*): There is no i such that for all elements in the domain of the SWFL, $xP_i y \rightarrow xPy$.

CONDITION $\bar{p}$: (*Weak Pareto Principle*): If for all i, $xP_i y$, then for all elements in the domain of the SWFL, consistent with this, we have xPy.

CONDITION $\bar{C}$: (*Cardinality)*: For each i, every possible linear transformation of any element of L_i belongs to L_i.

CONDITION $\bar{M}$: (*Non-Comparability*): For any L, the social ordering R yielded by the SWFL for each $W \in L$ must be the same.

Given these conditions applicable to SWFL we can have the following:

Theorem: *There is no SWFL satisfying the conditions* $\bar{U}, \bar{I}, \bar{D}, \bar{P}, \bar{C}$ *and* $\bar{M}$.

Proof: Let us consider a pair $x, y \in X$. For $W \in L$, we have $W_i(x)$ and $W_i(y)$ for all i. Now let L gets transformed into $\hat{L}$ through a change in the individual welfare function keeping the individual orderings the same. Clearly then, by condition $\bar{C}$, which gives us two degrees of freedom for the welfare measure for each person, we can find a $\hat{W} \in \hat{L}$, such that $W_i(x) = \hat{W}_i(x)$ and $W_i(y) = \hat{W}_i(y)$. By condition $\bar{I}$, $xRy \leftrightarrow x\hat{R}y$, where R and $\hat{R}$ are social orderings corresponding to W and $\hat{W}$. Hence by $\bar{M}$, the social ordering must be same for the elements of L as for those of $\hat{L}$. Thus the only possible SWFLs satisfying conditions $\bar{I}$ and $\bar{C}$ are all SWFs, with R a function merely of the n-tuples of individual orderings $(R_1, R_2, \ldots\ldots R_n)$. But we know that no SWF satisfies conditions U, I, D and P, which conditions are implied by conditions $\bar{U}, \bar{I}, \bar{D}$ and $\bar{P}$ for SWFL. The proof is then complete.

With the incorporation of ordinal non-comparability as well as cardinal non-comparability the above theorem can be restated as:

Theorem (a): *There is no SWFL satisfying the conditions ON,* $\bar{U}, \bar{I}, \bar{D}, \bar{P}, \bar{C}$ *and* $\bar{M}$.

Theorem (b): *There is no SWFL satisfying the conditions CN,* $\bar{U}, \bar{I}, \bar{D}, \bar{P}, \bar{C}$ *and* $\bar{M}$.

It is to be noted here that the loss of information induced by ruling out interpersonal comparisons is sufficient to precipitate the impossibility result, even without ruling out cardinal welfare information. However the remaining conditions are necessary for the impossibility in the sense that the removal of any one of them makes it possible to have a SWFL satisfying the rest of the conditions.

If we assume Non-Comparability we need not impose any restriction on L. Hence Non-Comparability holds if and only if $L = \bar{L}$. Let $\bar{L}$ under Non-Comparability is denoted by $\bar{L}(0)$. Full Comparability holds if W^* being any element of $\bar{L}$ implies that $\bar{L}$ includes only and all W such that for all i $W_i = \psi(W^*)$; ψ being any increasing function. Let $\bar{L}$ under Full Comparability is denoted by $\bar{L}(F)$. This Full Comparability bears the implication that there exists a one-to-one correspondence between the welfare functions of different individuals. Unit Comparability holds if W^* being any element of $\bar{L}$ implies that $\bar{L}$ includes only and all W such that for all i $W_i = a_i + bW_i^*$. Let $\bar{L}$ under Unit Comparability is denoted by $\bar{L}(1)$. This Unit Comparability bears the implication that the welfare function of one individual specifies a one-parameter family of welfare functions for every other individual each member of the family differing from any other by a constant. Level Comparability holds if W^* being any element of $\bar{L}$ implies that $\bar{L}$ includes exactly all W such that for any i, j and $x, y \in X$; $W_i^*(x) \geq W_j^*(y)$ if and only if $W_i(x) \geq W_j(y)$. Let $\bar{L}$ under Level Comparability is denoted by $\bar{L}(L)$.

It may be noted that Full Comparability makes interpersonal comparability just as "Full" as the measurability of individual welfares will allow. Thus with ordinal individual welfare functions, the comparability will not extend beyond level comparability, but with cardinal individual welfare functions, units will be comparable. In case of Partial Unit Comparability $\bar{L}$ is such that $\bar{L}(1) \subseteq \bar{L} \subseteq \bar{L}(0)$. Similarly in case of Partial level Comparability $\bar{L}$ is such that $\bar{L}(L) \subseteq \bar{L} \subseteq \bar{L}(0)$. Now for any $\bar{L}$, that is for every possible

assumption of interpersonal comparability the binary relation of aggregation, R^a, is a quasi ordering and Pareto criterion, R^p, is a sub relation of R^a. With Non-Comparability $R^a = R^p$ and with Unit Comparability or with Full Comparability R^a is a complete ordering.

Proof :

Reflexivity of R^a follows directly from each W_i being an order preserving transformation of R_i for every element of L. Transitivity of R^a is also immediate.

For any $(x, y, z) \in X$;

$xR^a y$ and $\quad yR^a z \rightarrow \sum_i [W_i(x) - W_i(y)] \geq 0 \quad$ and $\quad \sum_i [W_i(y) - W_i(z)] \geq 0 \quad$ for $\quad$ all $W \in \bar{L}$.

$\rightarrow \sum_i [W_i(x) - W_i(z)] \geq 0 \quad$ for all $W \in \bar{L}$.

$\rightarrow xR^a z$.

Again for any $(x, y) \in X$;

$xR^p y \to \forall i [W_i(x) - W_i(y)] \geq 0$ for every $W \in L$.

$\to xR^a y$ since $\bar{L} \subset L$.

In order to show that with Non-Comparability $R^a = R^p$ we are to show that $xR^a y \to xR^p y$. For any $x, y \in X : xR^p y \to \exists j : yP_j x \to \exists j : [W_j(y) - W_j(x)] > 0$ for every $W \in L$. For each W let us define $\alpha_1(W) = W_j(y) - W_j(x)$ and $\alpha_2(W) = \sum_{i \neq j} [W_i(x) - W_i(y)]$. Taking any arbitrary $W^* \in \bar{L}$ we note that if $\alpha_1(W^*) > \alpha_2(W^*)$ then clearly $\square \, xR^a y$. If $\alpha_1(W^*) < \alpha_2(W^*)$ then considering $W^{**} \in L$ such that $W_i^{**} = W_i^*$ for $i \neq j$ and $W_j^{**} = nW_j^*$ where n is any real number greater than $[\alpha_2(W^*) \div \alpha_1(W^*)]$ we get $\alpha_1(W^{**}) > \alpha_2(W^{**})$ and $W^{**} \in L$. Since $\bar{L} = L$, given Non-Comparability, we have $\square \, (xR^a y)$.

In order to show that R^a is a complete ordering with unit comparability or with full comparability let us assume that $W^* \in \bar{L}$ for any $x, y \in X$. Obviously $\sum_i [W_i^*(x) - W_i^*(y)] \geq 0$ or ≤ 0. Since for every $W \in \bar{L}$, for each $i, W_i = a_i + bW_i^*$, for some $b > 0$, we must have $\sum_i [W_i(x) - W_i(y)]$ either non-negative for each $W \in \bar{L}$ or non-positive for each $W \in \bar{L}$. Hence R^a must be complete. Since full comparability implies that $\bar{L}$ is even more restricted clearly R^a must be complete also in this case.

Given this background of various types of Comparability we can easily define a Comparison set $\bar{L}$ such that for every element W of $\bar{L}$ we declare that the pair of alternative social states have at least the same aggregate Welfare, that is, the sum of the welfare differences between pair of alternative social states is non-negative. Since for the purpose of aggregation we are really interested in the welfare Units and not in the respective origins it is convenient to specify the set of vectors B of coefficients of individual welfare measures with respect to any comparison set.

With unit comparability B is an open half line with origin 0, but excluding 0. The precise specification of the half line from origin 0 will depend on the element W^* chosen for the representation. On the other hand with non-comparability B will equal the entire non-negative orthant except the boundary. Actually the coefficient set of $\bar{L}$ with respect to W^* consists exactly of all vectors b such that some $W \in \bar{L}$ can be expressed as $W_i = a_i + b_i W_i^*$. With a general definition of partial comparability any B from a half line to the entire positive orthant falls in this category. However it would be reasonable to assume that B under partial comparability will satisfy certain regularity conditions. First, the coefficients should be scale independent. Second, it seems reasonable to assume the convexity of B. Third, the coefficients set obeys the regularity Axiom that for every possible partition of the set of individuals into subsets j and $k : B^2 \subset B^1$ and $(B^1 \not\subset B^2) \to \exists (b^1 \in B^1 and b^2 \in B^2) : [\forall i \in j : b_i^2 < b_i^1]$ and $[\forall i \in k : b_i^2 > b_i^1]$ so that $B^2 \subset B^1$ implies that aggregation quasi ordering R_1 with respect to B^1 is a sub-relation of R_2 with respect to B^2. This regularity Axiom can be viewed as a condition of symmetry but of a mild kind. A somewhat more demanding condition is the following.

Weak Symmetry Axiom: Each B is a convex polyhedral cone defined by $B = [b \mid \forall i, j : (b_i / b_j) \leq \beta_{ij} \geq 1]$, except the origin, and for any pair $B^1 and B^2$, $[\exists i, j : \beta_{ij}^1 > \beta_{ij}^2] \rightarrow [\forall i, j : \beta_{ij}^1 > \beta_{ij}^2]$. This is a much stronger requirement than the regularity Axiom. With the latter it is sufficient that any ray in $B^2 \subset B^1$ be an interior ray of B^1, whereas with weak symmetry every ray in B^2 has to be interior in B^1, if B^2 is a proper subset of B^1. When the extent of comparability is relaxed for any pair of individuals, it has to be relaxed for every pair of individuals, in case of weak symmetry. The ethical acceptability of the axiom depends on the appeal of directional symmetry between pairs and between each individual in a pair. It is to be noted that weak symmetry implies the regularity and we have thus a sequence of aggregation quasi-orderings, each a sub relation of the next, starting from the Pareto quasi-ordering, which is yielded by non-comparability, and ending up with a complete ordering, which is yielded by unit comparability. In between lie all the cases of partial comparability. As the extent of partial comparability is raised the aggregation quasi-ordering gets extended without ever contradicting an earlier quasi-ordering obtained for a lower extent of partial comparability.

A measure of degree of partial comparability $d(B)$ is useful here and can be defined as the arithmetic mean of comparability ratios for every ordered pair of individuals where a comparability ratio C_{ij} is defined as $C_{ij} = \inf(b_i / b_j) / \sup(b_i / b_j), b \in B.$ Since C_{ij} must lie within the closed interval [0,1], $d(B)$ is also defined over this interval. Further the following theorem holds.

Theorem: *Given convexity, scale independence, and weak symmetry, $d(B) = 0$ implies that the aggregation quasi-ordering will be the same as the Pareto quasi-ordering, and $d(B) = 1$ implies that it will be a complete ordering. Further, if $d(B^2) > d(B^1)$, the aggregation quasi-ordering R^1 will be a sub-relation of the aggregation quasi-ordering R^2.*

Proof: If $d(B) = 1$, clearly $C_{ij} = 1$ for each ordered pair i, j. In this case B will consist of only one ray through the origin, and unit comparability will hold. In fact R^a will then be a complete ordering. If, on the other hand, $d(B) = 0$, each C_{ij} must equal zero, so that the ratio b_i / b_j can be varied without bound for every i, j. This implies that non-comparability holds and R^a will equal the Pareto quasi-ordering R^p.

If $d(B^2) > d(B^1)$, then for some i, j, $C_{ij}^1 < C_{ij}^2$. This implies that for some pair i, j, either $\sup(b_i^1 / b_j^1) > \sup(b_i^2 / b_j^2)$ or $\inf(b_i^1 / b_j^1) < \inf(b_i^2 / b_j^2)$. If the former, then it follows from the Weak Symmetry Axiom that B^2 is a proper subset of B^1. If the latter, then $\sup(b_j^1 / b_i^1) > \sup(b_j^2 / b_i^2)$, and once again B^2 is a proper subset of B^1. If $B^2 \subset B^1$, then for all $x, y \in X : xR^1 y \rightarrow xR^2 y$ and as the Regularity Axiom holds this $B^2 \subset B^1$ implies that R^1 is a sub-relation of R^2. Since the Weak Symmetry Axiom implies the Regularity Axiom, R^1 must be a sub-relation of R^2. From the theorem it is clear that if the Axiom of Weak Symmetry holds, in addition to the assumptions of convexity and scale independence, then all cases of partial comparability can be measured by a precise degree, $d(B) = q$, of partial comparability. This degree of partial comparability is a real number lying in the closed interval [0,1] and

the corresponding quasi ordering R^q is a sub-relation of all quasi orderings obtained with higher degrees of partial comparability, i.e., for $d(B) > q$, while all quasi orderings obtained with lower degrees of partial comparability, i.e., for $d(B) < q$, are sub-relations of R^q. This monotonicity property in the relation between the continuum of degrees of comparability in the interval $[0,1]$ and the sequence of aggregation quasi-orderings from the Pareto quasi-ordering to a complete ordering is a phenomenon of interest. It should be noted that it is not necessary to assume $d(B) = 1$ for a complete ordering to be generated, though it is sufficient. Even with $d(B) < 1$, completeness may be achieved. The necessary degree depends on the precise configuration of individual welfare functions. If we assume Strong Symmetry Axiom, which states that there exists some functional combinations $W^* \in \bar{L}(P)$ such that for each $B(W^*, \bar{L})$, $Sup_{b \in B}(b_i / b_j)$ is exactly the same for all ordered pairs, i, j, we have the following theorem:

THEOREM: *With Convexity, scale independence, and Strong Symmetry the aggregation quasi-orderings will be complete if the degree of partial comparability is greater than or equal to $(\alpha^*)^2$, where*

$$\alpha^* = Sup_{x,y \in X} \alpha(x, y);$$

$$\alpha(x, y) = \min[m(x, y), m(y, x)] / \max[m(x, y), m(y, x)];$$

$$m(x, y) = \sum_{i \in j}[W_i^*(x) - W_i^*(y)];$$

$$m(y, x) = \sum_{i \in k}[W_i^*(y) - W_i^*(x)].$$

Proof: For any pair x, y, completeness will fail to be fulfilled if and only if $[W_i(x) - W_i(y)] > 0$ for some $W \in \bar{L}$ and < 0 for some other $W \in \bar{L}$. First let us consider W^*. Without loss of generality, let $\sum_i [W_i^*(x) - W_i^*(y)] > 0$, i.e., $m(x, y) > m(y, x)$. we have to show that the sum of welfare differences between x, y is non-negative for all $W \in \bar{L}$. Let the degree of partial comparability be d, so that the ratio of the welfare units of any two individuals can be reduced at most by a factor $p = d^{1/2}$. If the sum of welfare differences between x and y is negative for any $W \in \bar{L}$, $[pm(x, y) - m(y, x)] < 0$. Hence $p < [m(y, x) / m(x, y)]$. But this is impossible, since $d = p^2 \geq (\alpha^*)^2$ and $\alpha^* = Sup_{x,y \in X} \alpha(x, y)$. This contradiction proves that the aggregation quasi-ordering must be complete.

Professor Sen's Social Welfare Functional (SWFL) approach involves consideration of real valued welfare functions defined over a set of alternative social states and formation of comparison sets based on non-negative sum of welfare differences. If welfare functions are considered to be real valued functions and comparison sets are thus formed, a sequence of quasi-orderings leading to a complete ordering over all possible quasi-orderings are made possible under Weak Symmetry Axiom which imposes a directional symmetry between each individual in a pair and between pairs.

The ethical acceptability of the Axiom depends on the appeal of such directional symmetry. From the point of view of theoretical analysis to the framework of interpersonal comparability Prof. Sen's contribution may be worth mentioning. But from the practical point of view his SWFL approach is

highly subjected to criticism for using real valued welfare functions as its base and utilizing comparison sets as well as Weak Symmetry Axiom as its superstructure. The assumption of real valued welfare functions bears the implication that positive monotonic trans-formation and positive linear transformation of welfare indices belong to the same set and are real numbers. Alternative measurability-comparability conditions used by Prof. Sen and conclusions derived there from are purely mathematical truisms. They hardly correspond to the reality, as in reality we rarely observe such regularity among actual welfare position of individuals. Further interpersonal comparison becomes hardly possible due to variations amongst individuals in a given social state with respect to non-welfare indices such as number of dependants, job satisfaction, extent of disturbance in family life, access to nurture his/ her hobbies and so on. If one goes on making a list of such non-welfare indices list may include innumerable items. Consideration of all these items will make interpersonal comparison rarely possible.

Prof Sen has tried to provide an explanation to his analysis on partial comparability on the basis of informational set up and his conclusions are quite consistent with the common belief that degree of perfection in comparability varies directly with the availability of information. In one sense this common belief seems to be realistic. But there is another story where wider informational set up can be held responsible to make inter personal comparison hardly possible if not impossible.

In much of welfare economics and the theory of social choice, welfares of individuals in the society are assumed to be the sole basis of judgments about social welfare and social choice. This predominant reliance on individual welfares as the basis of social welfare evaluations constitutes a manifestation of 'welfarism'. Despite this dominance of welfarism there have been several important developments in welfare economics where departures from welfarism have been made by bringing in non-welfare information as essential ingredients for social evaluation of alternatives. As early as 1959, Prof. Hicks warned welfare economists against the sterility that may result from a rigid adherence to welfarism in normative economics. It is now clear that welfare economists have taken Hicks' warning seriously and they have shown an increasing interests to venture beyond the rigid boundaries set by the belief that social welfare judgments should be based on considerations of individual welfares. In the process welfare economists seek to widen the basis of social evaluation by bringing in non-welfare information along with the available information regarding individual welfares and both welfare and non-welfare information taken together constitute the basis of social evaluation of alternatives. As a result welfare economics requires a re-thinking.

Incorporation of non-welfare information into the analysis of social welfare evaluation leads to have expansion of informational base. Such widening of informational base is expected to result in directional asymmetry between each individual in a pair and between pairs. This in turn will break down the possibility of aggregation ordering to be quasi-ordering and/or complete ordering. As a result interpersonal comparison will be hardly possible. Conclusions derived without paying attention to those non-welfare indices are expected to provide wrong indication regarding actual welfare position of individuals concerned. Hence a rational method of social welfare evaluation must incorporate non-welfare information into the analysis of welfare information for successful evaluation of alternative

social states. In the process of such incorporation of non-welfare information we will hardly find a directional symmetry between each individual in a pair as well as between pairs.

In order to form a comparison set for full comparability there must exist a one to one correspondence between welfare functions of different individuals. In real life situation such correspondence is hardly found due to the existence of variations amongst individuals in a given social state with respect to non-welfare indices. These variations amongst individuals in a given social state with respect to non- welfare indices restrict us to have a one to one correspondence between welfare functions of different individuals. Coming to the case of unit comparability we note that unit comparability requires the specification of a family of welfare functions for every other individual each member of the family differing from any other by a constant provided that welfare function of one individual is already specified. In real life situation such systematic specification of a family of welfare functions for every other individual is hardly found due to the existence of variations amongst individuals in a given social state with respect to non-welfare indices. These variations amongst individuals in a given social state with respect to non- welfare indices restrict us to have such systematic specification of a family of welfare functions for every other individual from a specified welfare function of one individual. On the other hand ordinal level comparability requires that any individual welfare function be a monotonic transformation of every other individual welfare function. In real life situation such systematic correspondence between individual welfare functions are hardly found. It is due to the very existence of variations amongst individuals in a given social state with respect to non-welfare indices. These variations amongst individuals in a given social state with respect to non- welfare indices restrict us to have such systematic correspondence between individual welfare functions. In order to observe any systematic correspondence between individual welfare functions what we need is to assume that there exists a regularity condition among individual utility functions defined over alternative social states as well as among individual variations in respect of non-welfare indices. Though a regularity condition among real valued individual utility functions defined over alternative social states may be observed it will be highly unrealistic to expect that there exists a regularity condition among individual variations in respect of non-welfare indices.

Moreover aggregation of non-welfare indices constitutes another major problem in social welfare evaluation. So far it is recognized that several non-welfare indices like rights, liberties, cultural background, educational background etc. have their individual bearing on the level of welfare enjoyed by individuals. But practical problem lies with the identification of various non-welfare indices and evaluation of their impact on social welfare. The existing literature surveys how the theoretical framework of social welfare evaluation fails to capture the effects of an individual non-welfare index. Actually we do not have any systematic evaluation procedure by which we can capture the effects of all conceivable non-welfare indices. The entire problem of value judgment is the problem of attaching relative weightage to gainers as well as losers. There is no unique criterion of attaching such weightage. It varies from evaluator to evaluator depending on his personal judgment. Each evaluator then tries to provide justification behind his judgment. But the existence of convincing justification to the weightage attached will remain a far cry, as it is purely the personal judgment. This personal judgment is again

subjected to general acceptability, as consensus regarding such personal judgment can not be emerged. Ultimate result then is to have Non-Comparability among different distinct social states and it is the hard reality that we face in real life situation. There is no way out.

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