IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)

An International Open Access, Peer-reviewed, Refereed Journal

Infinitesimal Variation Of The Metric Tensor Induced On The Cross-section Of Co-tangent Bundle $C_{T(M)}$

Dr. Sudarshan Kumar

PGT (MATHS)

R. S.S. +2 School Mahna, Begusarai

ABSTRACT

Differential Geometry of fibred spaces with projectable geometrical objects there has tremendous manifestations in Mathematics.

We shall study the infinitesimal on the cross-section of cotangent bundle $C_{T(M)}$ by complete lift, horizontal lift and intermediate lift on $C_{T(M)}$ of an affine connection ∇ on M. Moreover, we shall also study the infinitesimal variation of the induced metric tensor on the cross-section of $C_{T(M)}$ by Tondeur-sato metric g^T on the co-tangent bundle $C_{T(M)}$ of M.

KEY WORDS : CT(M) = Co-tangent bundle; $^*\bar{G}_{ij}$; $\bar{G}\Lambda\mu$

INTRODUCTION

Let M be an n-dimensional differentiable manifold of class C^∞ and $C_T P$(M): the co-tangent space at a point P in M. i.e., the set of all covariant vectors of M at P. Then the set

$$C_{T(M)} = \underset{P \in M}{U} C_{T_{P(M)}}$$

is defined as co-tangent bundle over the manifold M. Any point ∅ of $C_{T(M)}$ such that P∈ $\boldsymbol{C_{T_{P(M)}}}$ is a convector at P in M.

The mapping π: $C_{\mathbf{T(M)}} \rightarrow$ M is a projection map from $C_{T(M)}$ on to M. $\pi^{-1}(P) = C_{T_{P(M)}}$ is called the fibre over P∈M.

COMPLETE LIFTS OF TENSOR FIELDS TO $C_{T(M)}$:

Suppose that there is a given vector field X in M. with components X^h. The complete lift X^c of X on M to $C_{T(M)}$ is a vector field on $C_{T(M)}$ with components (127).

$$x^C: \begin{bmatrix} X^h \\ -P_\alpha \partial_h X^\alpha \end{bmatrix}$$

With respect to the co-ordinate system (x^h, P_h).

Let F be a tensor field of type (1, 1) on M with components F_i^h and let it be complex structure on M. then, the complete lift F^C of F on M to $C_{T(M)}$ is a tensor field of type (1,1) on $C_{T(M)}$ with components

$$F^C: \begin{bmatrix} F^h & 0 \\ P_\alpha(\partial_i F_h^\alpha - \partial_h F_i^\alpha) & F_h^i \end{bmatrix}$$

THEOREM 1:

The necessary and Sufficient condition that on infinitesimal variation of the cross-section of co-tangent bundle $C_{\mathbf{T(M)}}$ associated with induced metric tensor $^*\bar{G}_{ij}$ due to complete lift ∇^C on $C_{\mathbf{T(M)}}$ to be isometric is that

$\nabla_i V_j + \nabla_j V_i - 2(\nabla_i \nabla_j V_k + V_\alpha R_{kij}^\alpha) V^k = 0$

PROOF :

Assume that the infinitesimal variation of the cross-section of CT(M) endowed with metric $^*\bar{G}_{ij}$ be isometric the $\delta^*\bar{G}_{ij} = 0$. As a result of (4.9.15), we have

$\nabla_i V_j + \nabla_j V_i - 2(\nabla_i \nabla_j V_k + V_\alpha R_{kij}^\alpha) V^k = 0$

Conversely if $\nabla_i V_j + \nabla_j V_i - 2(\nabla_i \nabla_j V_k + V_\alpha R_{kij}^\alpha) V^k = 0$

$\Rightarrow \quad [\nabla_i V_j + \nabla_j V_i - 2(\nabla_i \nabla_j V_k + V_\alpha R_{kij}^\alpha) V^k] = 0$

G being infinitesimal

⇨ $\delta^{*}\bar{G}_{ij} = 0$: ⇨ the variation of the cross-section of the cotangent bundle $C_{T(M)}$ endowed with metric induced by ∇_C is isometric.

THEORM 2:

The necessary and sufficient condition for a normal variation of the cross-section of $C_{T(M)}$ associated with the induced metric tensor ${}^{*}\bar{G}_{ij}$ due to that complete lift ∇^C on $C_{T(M)}$ to be isometric is that

$$\nabla_i\nabla_j V_k + V_\alpha R^{\alpha}_{kji} = 0$$

i.e, the cross-section of $C_{T(M)}$ by the induced metric tensor ${}^{*}\bar{G}_{ij}$ is geobesic with respect to the normal variation.

PROOF :

when V^{α} =0, i.e., when the variation vector V^{α} is normal to the cross-section $C_{T(M)}$,then the variation is called normal from the result that the normal variation of the cross-section $C_{T(M)}$ will be isometric iff

$$\nabla_i\nabla_j V_k + V_\alpha R^{\alpha}_{kij} = 0$$

THEOREM 3 :

In order for a variation of the cross- section of $C_{T(M)}$associated with the induced metric tensor ${}^{*}\bar{G}_{ij}$ due to the completed lift ∇^C on $C_{T(M)}$ to be conformal (homothetic), it is necessary and sufficient then

$$\nabla_i\nabla_j + \nabla_j V_i - 2\left(\nabla_i\nabla_j V_k + V_\alpha R^{\alpha}_{kji}\right)V^k = 2\kappa\, {}^{*}\bar{G}_{ij}$$

Where κ being a certain function (constant).

PROOF :

A variation of the cross-section of $C_{T(M)}$ associated with the induced metric tensor ${}^*\bar{G}_{ij}$ due to the complete lift ∇C on $C_{T(M)}$ is said to de conformal (homothetic) iff

$$\delta^*\bar{G}_{ij} = \lambda^*\bar{G}_{ij}$$

Consequently,

from the result

$$\delta^*\bar{G}_{ij} = \nabla_i\nabla_j + \nabla_j V_i - 2(\nabla_i\nabla_j V_k + V_\alpha R^\alpha_{kji})V^k$$

we reach the conclusion that

$$2\lambda^*\bar{G}_{ij} = \nabla_i\nabla_j + \nabla_j V_i - 2(\nabla_i\nabla_j V_k + V_\alpha R^\alpha_{kji})V^k$$

$$\Rightarrow 2\lambda^*\bar{G}_{ij}/G = \nabla_i\nabla_j + \nabla_j V_i - 2(\nabla_i\nabla_j V_k + V_\alpha R^\alpha_{kji})V^k$$

Where $2\lambda/\gamma$ to either a certain function or a constant.

CONCLUSION:

Finally we say that infinitesimal variation of the metric tensor induced on CT(M) by complete lift ∇C of an affine connection ∇ on M.

REFERENCES:-

1. Atlay, S. (2005) : Some application on weakly pseudosymmetric Riemannian Manifolds, Differential Geometry, Vol-7, 1-10.
2. Blair, D.E (1971) : Almost contact manifold with killing structure tensors, Pacific. J. Math., 39 (2), 285-292.
3. Atanasiu, G. and Mirela Tarnoveanu (2006) : Linear connection on the cotangent bundle of order two (T*2M, II*2, M) Differential Geometry- Dynamical systems, Vol-8, 8-22.
4. Carfi, D. (2004) : Tangent spaces an S-manifolds, differential Geometry-Dynamical System, Vol 6, 1-13.
5. Brickell, F. and R.S. Clark (1970) : Differentiable Manifold, an introduction, van Nostran Reinhold, Princeton, New York.

6. Hussain, S.I. and A. Shasrfuddin (1976) : Some symmetric metric connections in almost contact manifolds, Tensor, N.S., 30, 213-218.
7. Houch, C.S., and S. Ishihara (1972) : Tensor fields and connection on a cross-section in the tangent bundle of order, Kodai Math. Sem. Rep., 24, 234-250.
8. Halgason, S. (1962) : Differential geometry and symmetric spaces, Acad. Press. N.Y.
9. Yano, K. and E.M. Patterson (1967) : Vertical and complete lift from manifold to its cotangent bundle, J. Math. Soc. Japan, 19, 91-113
10. Hicks, N.J. (1965) : Notes on differential geometry. D. Van Nostrand Co., N.Y.
11. ____________(1967) : Horizontal lift from a manifold to its cotangent budle, J. Math. Soc. Japan, 19, 185-198.
12. Mok, K.P. (1977) : Metrices and connections on the cotangent bundle, Kodai Math., Sem., Rep…, 28, 226-238.