IJCRT.ORG ISSN : 2320-2882

INTERNATIONAL JOURNAL OF CREATIVE RESEARCH THOUGHTS (IJCRT)
An International Open Access, Peer-reviewed, Refereed Journal

HYDRODYNAMIC FLOW NEAR A TIME-VARYING ACCELERATED POROUS PLATE IN ROTATING SYSTEM

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ABSTRACT

An initial Value investigation has been made of an incompressible viscous homogeneous fluid over a porous plate with uniform suction or blowing. A solution describing the general feature of the unsteady hydrodynamic boundary layer flow in a rotating fluid with suction (blowing) has been derived. It is assumed that both the plate and the fluid are in state of solid body rotation with constant angular velocity Ω about Z-axis normal to the plate and the plate is supposed to be accelerated with a given velocity. The results thus obtained have been compared with previous investigations on the topic. Some particular cases of interest have been discussed. Special care has been paid to the physical interpretation of mathematical results obtained for steady state solution. The significant effect of the suction parameter on the flow phenomenon in this case has also been investigated.

KEY WORDS - Viscous - incompressible, suction, Blowing, unsteady flow, Accelerated porous plate, Boundary layer.

1. INTRODUCTION:

The flow of a viscous incompressible and electrically conducting fluid past an impulsively moving infinite plate in the presence of an external magnetic field has been investigated by Rossow (1957) and Kukutani (1958). They studies the problems in the case of infinite plate with and without the induced magnetic field by the current. Further Gupta (1960), Nanda and Sundram (1962), Soundalgekar (1965), Pop (1967), Mohapatra (1971), Mishra and Mohpatra (1973) extended the problem.

Gupta (1972) has studies an exact solution for the flow past a plate with uniform suction in a rotating reference frame. Debnath and Mukherjee (1973) have investigated the motion of incompressible homogeneous, viscous fluid bounded by porous plates with uniform suction (blowing). They considered both the fluid and the plate in a state of a solid body rotation with constant angular velocity Ω about z-axis normal to the plate. They imposed an additional non-torsional oscillation of a given frequency ω on the plate for the generation of unsteady flow in a rotating system.

In the present paper the problem of Debnath and Mukherjee (1973) under different conditions has been reviewed.

An initial value investigation has been made of the motion of an incompressible, homogeneous, viscous fluid over a porous plate with uniform suction or blowing. Both the plate and the fluid are in the state of a solid body rotation with constant angular velocity about z-axis normal to the plate and the plate is assumed to be accelerated with a given velocity. A solution describing the general feature of the unsteady hydrodynamic boundary layer flow in a rotating system with suction (blowing) has been obtained. The results obtained have been compared with previous investigations on the topic. Some particular cases of interest have been discussed. Special attention has been made to the physical interpretations of mathematical results obtained for steady state solution.

The significant effect of the suction paramater on the flow phenomenon has also been investigated in this paper.

2. MATHEMATICAL FORMULATION OF BASIC EQUATIONS:

The Navier-Strokes' equations and the equation of continuity for the unsteady motion of a viscous fluid in a rotating reference frame given by Debnath (1972) are

$$\frac{\partial \vec{u}}{\partial t'} + (\vec{u}.\nabla)\vec{u} + 2\Omega\vec{k} \times \vec{u} = \frac{1}{\rho}\nabla p + \upsilon\nabla^2\vec{u} \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2.1)}$$

$$\text{div } \vec{u} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2.2)}$$

where $\vec{u} = (u, v, w)$ is the velocity vector, $\vec{k}$ the unit vector along z-axis, p the pressure including centrifugal term, ρ the density of fluid, υ the kinematic viscocity. The velocity is assumed to be dependent on z and t, so that

$$\vec{u}(z, t) = [u(z, t), v(z, t), w(z, t)] \ldots\ldots\ldots\ldots \text{(2.3)}$$

It follows from equation (2.2), together with uniform suction (blowing) that W = $-W_0$ is constant. Clearly $W_0 > 0$ for suction and $W_0 < 0$ for blowing.

In absence of pressure gradient, the equations of motion (2.1) can be written as :

$$\frac{\partial u}{\partial t} - W_0\frac{\partial u}{\partial z} - 2\Omega v = \upsilon\frac{\partial^2 u}{\partial z^2} \quad \ldots\ldots\text{(2.4)}$$

$$\frac{\partial^2 v}{\partial t^2} - W_0\frac{\partial v}{\partial z} + 2\Omega u = \upsilon\frac{\partial^2 v}{\partial z^2} \quad \ldots\ldots\text{(2.5)}$$

Let us take $q = u + iv$. we multiply equation (2.5) by

$i = \sqrt{-1}$ and adding with equation (2.4) we obtain:

$$\frac{\partial}{\partial t}(u + iv) - W_0 \frac{\partial}{\partial z}(u + iv) + 2\Omega i(u + iv) = \upsilon \frac{\partial^2}{\partial z^2}(u + iv) \quad \text{i.e.} \quad \frac{\partial q}{\partial t} - W_0 \frac{\partial q}{\partial z} + 2\Omega i q = \upsilon \frac{\partial^2 q}{\partial z^2} \qquad \text{.........(2.6)}$$

The boundary conditions for the present problem

are

$q\ (z, t) = 0$ for all $z = 0$ and $t \leq 0$ ....(2.7)

$q\ (z, t) = \lambda e^{i\omega t}\, t^n$, $W = W_0$ for $z = 0$, and $t > 0$ ....(2.8)

and $q \to 0$ or infinite as $z \to \infty$ for $t > 0$ ....(2.9)

where λ is constant with the dimensions of velocity. The present boundary conditions are the generalisation of Debnath and Mukherjee (1973). They appear to have some physical importance regarding geophysical and astrophysical problems. They reveal some significant and interesting characteristic features of the hydrodynamic spin up flows which Debnath (1975) has pointed out in his investigations.

3. SOLUTION OF THE PROBLEM :

It is convenient to introduce non-dimentional variables and non-dimensional parameters respectively in the forms

$\eta = \frac{z\lambda}{\upsilon}$, $t = \Omega t'$, $U = \frac{q}{\lambda}$ ....(3.10)

and $R = \frac{W_0}{\lambda}$, $N = \frac{2\Omega\upsilon}{\lambda^2}$, $\sigma = \frac{\omega}{\Omega}$ ....(3.11)

then equation (2.6) and the boundary and initial conditions (2.7) - (2.9) become

$\frac{\partial^2 U}{\partial \eta^2} + R\frac{\partial U}{\partial \eta} - iN\,U = \frac{N}{2}\frac{\partial U}{\partial t}$ ....(3.12)

$U = 0$ everywheren for $t \leq 0$ ....(3.13)

$U = \lambda e^{i\sigma t} . t^m$ at $\eta = 0, t > 0$ ....(3.14)

and U = 0 or infinite as $\eta = \infty, t > 0$ ....(3.15)

In order to solve the initial value problem, we introduce the Laplace transform $\bar{U}(\eta,p)$ of U (η,t) defined by the integral

$\bar{U}(\eta, p) = \int_0^\infty e^{-pt} U(\eta, t)\, dt$ ....(3.16)

The Laplace transform of equation (3.12) and the boundary conditions (3.13) - (3.14) are given by

$\frac{d^2\bar{U}}{d\eta^2} + R\frac{d\bar{U}}{d\eta} - (iN + \frac{p}{2}N)\bar{U} = 0$ ....(3.17)

$\bar{U}(\eta, p) = \frac{\lambda\Gamma(m+1)}{(p - i\sigma)^{m+1}}$ at $\eta = 0$ ....(3.18)

and $\bar{U}(\eta, p) = 0$ or infinite as $\eta \to \infty$ ....(3.19)

Taking $\frac{d}{d\eta} \equiv$ D, equation (3.17) can be written as

$[D^2 + RD - (iN + p\frac{N}{2})]\bar{U} = 0.$

The auxiliary equation corresponds to

$D^2 + R D - (iN + \frac{pN}{2})] = 0$

$\therefore$ D =

$= -\frac{R}{2} \pm \sqrt{\frac{N}{2}\left(p + \frac{R^2 + 4iN}{2N}\right)}$.

Thus the solution of (5.3.17) is written as

$\bar{U}(\eta, p) = C_1 \exp\left[\frac{-R\eta}{2} + \eta\sqrt{\frac{N}{2}\left(p + \frac{R^2 + 4iN}{2N}\right)}\right]$

$+ C_2 \exp\left[\frac{-R\eta}{2} - \eta\sqrt{\frac{N}{2}\left(p + \frac{R^2 + 4iN}{2N}\right)}\right]$ ....(3.20)

Using boundary conditions (3.18) - (3.19) to equation (3.20) we obtain,

$\bar{U}(\eta, p) = \frac{\lambda\Gamma(m+1)}{(p - i\sigma)^{m+n}} \exp\left[\frac{-R\eta}{2} - \eta\sqrt{\frac{N}{2}\left(p + \frac{R^2 + 4iN}{2N}\right)}\right]$ .... (3.21)

Inverting equation (3.21) by Fourier-Mellin inversion integral

we get,

$U\_(\eta,t) = \lambda\Gamma(m+1)\, 2\pi i \quad \exp(-\eta R \quad 2) \int_{c-i\infty}^{c+\infty} \exp\ pt - \eta \quad N\, 2 .(p + R^2 + 4iN \quad 2N \quad)$

$dp \quad (p - i\sigma)^{m+1}$ ....(3.22)

Let us put

$Q = \eta \quad N\, 2 ,\ x^2 = p + R^2 + 4iN \quad 2N \quad ,\ \alpha = i\sigma + R^2 + 4iN \quad 2N \quad ,$

then we have

U $(\eta,t) = \lambda\Gamma(\mu + 1) \exp(- \eta R \quad 2)\ I(\eta,t,\alpha,m)$ ....(3.23)

where

$I(\eta,t,\alpha,m)$

$= 1 \quad 2\pi i \int_{Br^2} \exp\ x^2 - R^2 + 4iN \quad 2N \quad t - Qx\ 2x \quad dx \quad (x^2 - \alpha)$

...........(3.24)

the path Br_2 is Bromwich path defined in McLachlan (1963).

Now we have to find the values of $I(\eta,t,\alpha,m)$ for different values of m.

Let,

$F(\alpha) = I(\eta,t,\alpha,0)$

$= 1 \quad 2\pi i \int_{Br^2} \exp x^2 - R^2 + 4iN \quad 2N \quad t - Qx \quad 2x \quad (x^2 - \alpha)dx$

...........(3.25)

Differentiating (3.25) w.r.t. 'α' we get,

$I(\eta,t,\alpha,1) = dF \quad d\alpha + tF$ ....(3.26)

Again differentiating (3.26) successively $(m - 1)$ times w.r.t. 'α'

we get $I(\eta,t,\alpha,m)$ for different values of m.

Using McLachlan (1963), we have

$$I(\eta,t,\alpha,0) = \underline{1}\ 2\ \exp(i\sigma t)\exp(Q\sqrt{\alpha})\ erfc\ (\underline{Q + 2t\sqrt{\alpha}}\quad 2\sqrt{t}\ )$$

$$+ \exp(-Q\sqrt{\alpha})\ erfc\ (\underline{Q - 2t\sqrt{\alpha}}\quad 2\sqrt{t}\ ) \qquad ....(3.27)$$

With the help of equations (3.26) and (3.27) we get,

$$I(\eta,t,\alpha,1) = \underline{1}\ 2\ \exp(i\sigma t)\exp(Q\sqrt{\alpha})\ erfc\ (\underline{Q + 2t\sqrt{\alpha}}\quad 2\sqrt{t}\ )(\underline{Q}\quad 2\sqrt{\alpha} + 1)$$

$$- \exp(Q\sqrt{\alpha}) erfc\ (\underline{Q - 2t\sqrt{\alpha}}\quad 2\sqrt{t}\ )(\underline{C}\quad 2\sqrt{\alpha} - t) \quad ....(3.28)$$

Similarly we can get $I(\eta,t,\alpha,2)$ and in general $I(\eta,t,\alpha,m)$ for different integral values of m.

4. SOME SPECIAL CASES :

Case 1: When $i\sigma t \neq 0$, and m = 0, the velocity field is obtained with the help of equations (3.23) and (3.27) as

$$\underline{U(\eta + t)}\quad \lambda \quad = \underline{1}\ 2\ exp\ i\sigma t - \underline{R\eta}\ 2\ \exp\eta\ \underline{N}\ 2\ ^{1/2} i\sigma + \underline{R^2\ + 4iN}\quad 2N\quad ^{1/2}$$

$$x\ erfc\eta\ \underline{N}\ 2\ ^{1/2} + i\sigma + \underline{R^2\ + 4iN}\quad 2N\quad ^{1/2}\sqrt{t}$$

$$+ \exp{-\eta}\ \underline{N}\ 2\ ^{1/2}\ i\sigma + \underline{R^2\ + 4iN}\quad 2N\quad ^{1/2}$$

$$x\ erfc\eta\ 2\ \underline{N}\ 2t\ ^{1/2} - i\sigma + \underline{R^2\ + 4iN}\quad 2N\quad ^{1/2}\sqrt{t}$$

...........(4.29)

Where erfc (x) is the complementary error function by Lebedev (1965).

i.e.

$$erfc\ (x) = 1 - erf\ (x) = \underline{2}\quad \sqrt{\pi}\ \int_x^{\infty} e^{-\tau^2}\, d\tau \qquad ....(4.30)$$

The solution (3.29) describes the general feature of unsteady hydrodynamic boundary layer flow in rotating fluid including the effect of uniform suction (blowing) according as R > 0 or R < 0. This result agrees with the result of Gupta (1972), Debnath and Mukherjee (1973). By taking R = 0, the velocity distribution (4.29) is exactly identical with Thornely (1968).

Case II : when $i\sigma t \neq 0$, m = 1,

then the velocity distribution derived with the help of equations (3.23) and (3.28) is

$\underline{U(\eta, t)}\quad \lambda$

$\_\_\_\_\_ = \underline{1}\ 2\ exp\ i\sigma t\ \underline{R\eta}\ \ 2\ \ \exp\eta\ \underline{N}\ \ 2\ ^{½} i\sigma + \underline{R^2}\ \underline{+ 4iN}\quad 2N\quad ^{½}$

$x\ erfc\,\underline{\eta}\,2\ \underline{N}\ \ 2t\ ^{½} + i\sigma + \underline{R^2}\ \underline{+ 4iN}\quad 2N\quad ^{½}$

$x\ \ erfc\,\eta\ \underline{N}\ \ 2\ ^{½}\qquad\qquad +1$

$-\ \exp -\eta\ \underline{N}\ \ 2\ ^{½}\,.\ i\sigma + \underline{R^2}\ \underline{+ 4iN}\quad 2N\quad ^{½}$

$x\ erfc\ \underline{\eta}\,2\ \underline{N}\ \ 2t\ ^{½} - i\sigma\ +\ \underline{R^2}\ \underline{+ 4iN}\quad 2N$

$x\ \ \underline{\eta}\,2\ \underline{N}\ \ 2\ ^{½}\qquad\qquad -1$

...........(4.31)

Case III : when $i\sigma t \neq 0$, m = 2,

The velocity field, with the help of equations (3.23) and (3.26) reduces to

$U(\eta, t) = \lambda\ exp\ -\ \underline{\eta R}\ \ 2\ \ \underline{d}\ \ d\alpha\, I(\eta,t,\alpha,1) + tI(\eta,t,\alpha,1)$

...........(4.32)

Using equations (3.28) and (4.32), the velocity field is obtained as

$\underline{\quad U(\eta, t)}\quad \lambda t^2\quad = \underline{1}\ 2\ exp\ i\sigma t\ -\ \underline{\eta R}\ \ 2\ \ \exp\eta\ \underline{N}\ \ 2\ ^{½} i\sigma + \underline{R^2}\ \underline{+ 4iN}\quad 2N\quad ^{½}\ x$

$erfc\,\underline{\eta}\,2\ \underline{N}\ \ 2t\ ^{½} + i\sigma + \underline{R^2}\ \underline{+ 4iN}\quad 2N\quad .^{½}t^{½}$

x

-

$+\ \exp -\eta\ \underline{N}\ \ 2\ ^{½}\ i\sigma + \underline{R^2}\ \underline{+ 4iN}\quad 2N\quad ^{½} x\ erfc\ \eta\ \underline{N}\ \ 2t\ ^{½}$

$-\ i\sigma\ +\ \underline{R^2}\ \underline{+ 4iN}\quad 2N\quad ^{½}.t^{½}\ x$

$-\qquad\qquad +1\ x$

$-\ \underline{\eta\sqrt{N}}\quad \pi\sqrt{2}\ \ \exp\ \underline{-\eta^2\ N}\quad 8\ \ + i\sigma + \underline{R^2}\ \underline{+ 4iN}\quad 2N$

...........(4.33)

The asymptotic representation of the complementary error function as $t \to \infty$, the equation (3.29) reduces to steady state from

$\underline{\quad U(\eta, t)}\quad \lambda\quad = exp\ i\sigma t\ -\ \eta\underline{R}\ 2\ \ +\ \underline{N}\ \ 2\ ^{½}\ i\sigma + \underline{R^2}\ \underline{+ 4iN}\quad 2N\quad ^{½}$

...........(4.34)

which includes the effect of suction.

By setting R = -R_1, it follows $R_1 > 0$ and the result (4.34) reduces to the form

$$\frac{U(\eta, t)}{\lambda} = exp\, i\sigma t - \eta \left[-\frac{R_1}{2} + \frac{N}{2} \left\{ \frac{i\sigma + R^2 + 4iN}{2N} \right\}^{1/2} \right] \quad ....(4.35),$$

including the effect of blowing.

5 CONCLUSIONS AND REMARKS :

The equations (3.23) and (3.24) describe the general feature of the unsteady hydrodyamic boundary layer flow in rotating system. For $m = 0$, in particular, we have a pure oscillation of a given frequency ω imposed on the plate. The solution that we have obtained is in the close agreement with Debnath (1973). Equations (4.31) and (4.33) give the velocity distributions for special cases when $m = 1$ and $m = 2$ respectively. In particular, the steady state solution has been obtained when $i\sigma t \neq 0$, m = 0. The effects of suction and blowing on the steady state solution has been obtained graphically.

Fig. (1) shows the effect of suction on the steady state velocity profile. Here we have plotted $\frac{U(\eta, t)}{\lambda}$ against η for different values of suction parameter R = 0.1, 0.2 and 0.3 respectively. It is observed that the velocity decreases more rapidly with increase in suction parameter and consequently the boundary layer thickness is reduced with the increase in suction parameter.

Fig. (2) displays the effect of blowing on the flow for same values of σ, t, N ($\sigma = 0.1$, $t = 1$, $N = 0.001$). It has been plotted for different values of blowing parameter R_1 (R_1 = 0.1, 0.2 and 0.3 respectively). It is concluded that the blowing has a growing effect on the velocity profile and the velocity increases rapidly with the blowing parameter R_1.

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