



# Unique Fixed Point On Parametric Metric Space Employing C-Class Function

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**Abstract:** In this study, we employ the C-Class function to demonstrate a few fixed point conclusions on parametric metric spaces. Our findings expand upon and generalize the findings of Umashankar Singh and Naval Singh [23] in the context of novel rational contractive conditions. Additionally, some instances are given to highlight the key findings. This extension not only enhances the understanding of fixed point theory but also creates new opportunities for its application in more complex and diverse mathematical settings. Consequently, our research advances the field, offering a robust foundation for future studies and potential applications across various scientific and engineering disciplines.

**Keywords:** Parametric metric spaces, fixed point, C-class function.

## 1. Introduction:

Fixed point theories solve the existence and uniqueness of various types of problems for partial differential equations, integral equations, variational inequalities, approximate theory, game theory, and other fields. They are widely used in many branches of mathematics, engineering, economics, and medical sciences. Hussain et al. (2014) carried out the natural generalization of metric space, introducing the concept of parametric metric spaces and proving some fixed point results. Rao et al. (2014) introduced a novel form of generalized metric space known as parametric S-metric spaces. They also demonstrated several well accepted fixed point theorems under a range of expansion circumstances. Hussain et al. (2015) and Krishnakumar et al. (2016) expanded on these findings, introducing complete parametric b-metric spaces and providing

several instances to support several fixed point results. Subsequently, certain fixed point theorems for continuous and surjective expansion type mapping were proved by Daheriya et al. in 2016[5][6]. 2018 saw the introduction of parametric A-metric and parametric Nb-metric spaces by Priyobarta et al.[17] and Tas et al. [22], who also established the fixed-circle theorem for parametric Nb-Metric with some applications and certain fixed point results. Using C-class functions, Ege. O and De la Sen M.[7] expanded on the findings of Hussain et al.[10][11] in 2019 and established a few fixed point theorems for parametric metric spaces. Using the C-class function, U. Singh [23] established fixed point and common linked fixed point results in 2020. The fixed point theorems were established in 2022 by Garg A.K. et al.[9].

In this work, we expand and extend the results of Singh U. et al. [23] for new rational contractive conditions, and we show various fixed point results on parametric metric spaces using the C-class functions.

## 2. Preliminaries

To bolster our primary findings, we now review a few definitions and concepts related to parametric space:

**Definition 2.1[6]** Let  $X$  be a non-empty set and a  $\mathcal{F}_p: X \times X \times (0, \infty) \rightarrow [0, \infty)$  be a map on  $X$ , is said to be parametric metric on  $X$  if

- (a)  $F_p(x, y, t) = 0 \Rightarrow x = y$ , for all  $t > 0$
- (b)  $F_p(x, y, t) = F_p(y, x, t)$ , for all  $t > 0$
- (c)  $F_p(x, y, t) \leq F_p(x, z, t) + F_p(z, y, t)$ . for all  $x, y, z \in X$  and  $t > 0$

Then  $F_p$  is called parametric metric and the pair  $(X, \mathcal{F}_p)$  is called parametric metric space.

**Definition 2.2[6]** Let  $\{x_n\}_{n=1}^{\infty}$  is a sequence in parametric metric space  $(X, \mathcal{F}_p)$  then

- (i)  $\{x_n\}_{n=1}^{\infty}$  is called convergent to  $x \in X$  if  $\lim_{n \rightarrow \infty} F_p(x, x_n, t) = 0$  written as  $\lim_{n \rightarrow \infty} x_n = x$  for all  $t > 0$
- (ii)  $\{x_n\}_{n=1}^{\infty}$  is called Cauchy sequence in  $X$  if  $\lim_{n, m \rightarrow \infty} F_p(x_n, x_m, t) = 0$  for all  $t > 0$
- (iii) A parametric metric space  $(X, F_p)$  is called complete iff every Cauchy sequence is convergent to  $x \in X$ .

**Definition 2.3[6]** let  $(X, \partial)$  be a complete parametric metric space and a function  $\mathcal{F}_p: X \rightarrow X$  is a called continuous in  $x \in X$  if for any sequence  $\{x_n\}_{n=1}^{\infty}$  in  $X$  such that  $\lim_{n \rightarrow \infty} x_n = x$  for all  $t > 0$  then  $\lim_{n \rightarrow \infty} \mathcal{F}_p x_n = \mathcal{F}_p x$ .

**Example: 2.4[6]** Let  $X = \{f/f: (0, +\infty) \rightarrow R\}$  and defined a function  $\mathcal{F}_p: X \times X \times (0, \infty) \rightarrow [0, \infty)$  by  $F(K_1, K_2, t) = |K_1(t) - K_2(t)|$ , for all  $K_1, K_2 \in X$  and  $t > 0$  then  $F_p$  is a parametric metric in  $X$  and the pair  $(X, F_p)$  is called parametric metric space in  $X$

**Definition 2.5[3]** In a mapping  $\mathcal{F}_p: X \times X \rightarrow X$ , an element  $(x, y) \in X^2$  is called coupled fixed point of the if  $\mathcal{F}_p(x, y) = x$  and  $\mathcal{F}_p(y, x) = y$ , for  $x, y \in X$ .

**Example: 2.6** Let  $X = R$  and  $\mathcal{F}_p: X \times X \rightarrow X$  is defined by  $\mathcal{F}_p(a, b) = \frac{ab}{2}$

Since  $(0,0)$  is Coupled fixed point of  $\mathcal{F}_p$ .

**C-class function** has been defined in [1]

**Definition 2.7[1]** A continuous mapping  $F: (0, \infty) \times [0, \infty) \rightarrow R$  is said to be C-class function if it satisfies the following conditions:

$$\{C_a\} F(\eta, \varrho) \leq \eta, \text{ For all } \eta, \varrho \in [0, \infty).$$

$$\{C_b\} F(\eta, \varrho) \leq \eta \Rightarrow \text{Either } \eta = 0 \text{ or } \varrho = 0.$$

**Example: 2.8[1][2]** The following functions  $F: (0, \infty) \times [0, \infty) \rightarrow R$  are elements of C for all  $\eta, \varrho \in [0, \infty)$ ;

$$(I) \quad F(\eta, \varrho) = \eta - \varrho, F(\eta, \varrho) = \eta \Rightarrow \varrho = 0;$$

$$(II) \quad F(\eta, \varrho) = \eta - \varrho, F(\eta, \varrho) = \eta \Rightarrow \eta = 0 \text{ or } \varrho = 0;$$

$$(III) \quad F(\eta, \varrho) = \beta\eta, 0 < \beta < 1, F(\eta, \varrho) = \eta \Rightarrow \eta = 0;$$

$$(IV) \quad F(\eta, \varrho) = \eta - \frac{t}{k+t}, F(\eta, \varrho) = \eta = \varrho = 0;$$

$$(V) \quad F(\eta, \varrho) = \eta - \frac{2+t}{k+t}, F(\eta, \varrho) = \eta \Rightarrow \varrho = 0;$$

$$(VI) \quad F(\eta, \varrho) = \eta - \varphi(\eta), F(\eta, \varrho) = \eta \Rightarrow \eta = 0, \text{ here } \varphi: [0, \infty) \rightarrow [0, \infty) \text{ is a continuous function such that } \varphi(t) = 0 \Leftrightarrow t = 0;$$

$$(VII) \quad F(\eta, \varrho) = \eta\beta(\eta), \beta: [0, \infty) \rightarrow [0,1), \text{ and is a continuous function, } \\ F(\eta, \varrho) = \eta \Rightarrow \eta = 0;$$

Let  $\psi$  denote the set of all continuous and monotone non-decreasing function  $\phi: [0, \infty) \rightarrow [0, \infty)$  such that  $\phi(\varrho) = 0$  if and only if  $\varrho = 0$ ,  $\phi(\eta + \varrho) \leq \phi(\eta) + \phi(\varrho)$  for all  $\eta, \varrho \in [0, \infty)$ .

Let  $\Phi_1$  denote the all continuous function  $\phi: [0, \infty) \rightarrow [0, \infty)$  such that  $\phi(K) = 0$  if and only if  $\varrho = 0$  and  $\Phi_u$  denote the set of all continuous function  $\phi: [0, \infty) \rightarrow [0, \infty)$  such that  $\phi(0) \geq 0$ , note that  $\Phi_1 \subset \Phi_u$ .

Now we present our main results with new rational contractive conditions.

### 3. Main Results

**Theorem (3.1)** let  $(X, \partial)$  be a complete parametric metric space and  $\mathcal{F}_p: X \rightarrow X$  is a continuous

Mapping, satisfying the condition:

$$\phi \left( \partial(\mathcal{F}_p x, \mathcal{F}_p y, t) \right) \leq \mathbb{F} \left( \phi(\Omega(x, y)), \varphi(\Omega(x, y)) \right)$$

for all  $x, y \in X$ , for all  $t > 0$  and  $\mathbb{F} \in \mathbb{C}$ ,  $\phi \in \psi$ ,  $\varphi \in \Phi_u$ ,

$$\Omega(x, y) = a \text{ Max} \{ \partial(x, \mathcal{F}_p x, t), \partial(y, \mathcal{F}_p y, t) \} + b [ \partial(x, \mathcal{F}_p x, t) + \partial(y, \mathcal{F}_p y, t) ] \\ + c [ \partial(x, \mathcal{F}_p y, t) + \partial(y, \mathcal{F}_p x, t) ]$$

$$d \left[ \frac{\partial(x, \mathcal{F}_p x, t) \cdot \partial(\mathcal{F}_p x, y, t)}{\partial(x, \mathcal{F}_p x, t) + \partial(x, y, t)} \right] + e \left[ \frac{\partial(x, \mathcal{F}_p x, t) \cdot \partial(y, \mathcal{F}_p y, t)}{\partial(x, \mathcal{F}_p x, t) + \partial(x, y, t)} \right]$$

Where,  $2a + b + c + e < 1$  and  $a, b, c, d, e \in \left[0, \frac{1}{2}\right)$ . Then  $\mathcal{F}_p$  has a unique fixed point in  $X$ .

**Proof:** Choose  $x_0 \in X$  and define a sequence  $\{x_n\}_{n \in \mathbb{N}}$  as follows  $\mathbb{F}_p x_n = x_{n+1}$  for  $n = 1, 2, 3 \dots$  then take  $x = x_n$  and  $y = x_{n+1}$  in above equality then

$$\begin{aligned} &\phi(\partial(x_{n+1}, x_{n+2}, t)) = \phi(\partial(\mathbb{F}_p x_n, \mathbb{F}_p x_{n+1}, t)) \\ &\leq \mathbb{F} \left[ \begin{aligned} &\phi \left\{ \begin{aligned} &a \text{Max}[\partial(x_n, \mathbb{F}_p x_n, t), \partial(x_{n+1}, \mathbb{F}_p x_{n+1}, t)] + b[\partial(x_n, \mathbb{F}_p x_n, t) + \partial(x_{n+1}, \mathbb{F}_p x_{n+1}, t)] \\ &\quad + c[\partial(x_n, \mathbb{F}_p x_{n+1}, t) + \partial(x_{n+1}, \mathbb{F}_p x_n, t)] \\ &\quad + d \left[ \frac{\partial(x_n, \mathbb{F}_p x_n, t) \cdot \partial(\mathbb{F}_p x_n, x_{n+1}, t)}{\partial(x_n, \mathbb{F}_p x_n, t) + \partial(x_n, x_{n+1}, t)} \right] + e \left[ \frac{\partial(x_n, \mathbb{F}_p x_n, t) \cdot \partial(x_{n+1}, \mathbb{F}_p x_{n+1}, t)}{\partial(x_n, \mathbb{F}_p x_n, t) + \partial(x_n, x_{n+1}, t)} \right] \end{aligned} \right\} \\ &\varphi \left\{ \begin{aligned} &a \text{Max}[\partial(x_n, \mathbb{F}_p x_n, t), \partial(x_{n+1}, \mathbb{F}_p x_{n+1}, t)] + b[\partial(x_n, \mathbb{F}_p x_n, t) + \partial(x_{n+1}, \mathbb{F}_p x_{n+1}, t)] \\ &\quad + c[\partial(x_n, \mathbb{F}_p x_{n+1}, t) + \partial(x_{n+1}, \mathbb{F}_p x_n, t)] \\ &\quad + d \left[ \frac{\partial(x_n, \mathbb{F}_p x_n, t) \cdot \partial(\mathbb{F}_p x_n, x_{n+1}, t)}{\partial(x_n, \mathbb{F}_p x_n, t) + \partial(x_n, x_{n+1}, t)} \right] + e \left[ \frac{\partial(x_n, \mathbb{F}_p x_n, t) \cdot \partial(x_{n+1}, \mathbb{F}_p x_{n+1}, t)}{\partial(x_n, \mathbb{F}_p x_n, t) + \partial(x_n, x_{n+1}, t)} \right] \end{aligned} \right\} \end{aligned} \right] \\ &\leq \mathbb{F} \left[ \begin{aligned} &\phi \left\{ \begin{aligned} &a \text{Max}[\partial(x_n, x_{n+1}, t), \partial(x_{n+1}, x_{n+2}, t)] + b[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] \\ &\quad + c[\partial(x_n, x_{n+2}, t) + \partial(x_{n+1}, x_{n+1}, t)] \\ &\quad + d \left[ \frac{\partial(x_n, x_{n+1}, t) \cdot \partial(x_{n+1}, x_{n+1}, t)}{\partial(x_n, x_{n+1}, t) + \partial(x_n, x_{n+1}, t)} \right] + e \left[ \frac{\partial(x_n, x_{n+1}, t) \cdot \partial(x_{n+1}, x_{n+2}, t)}{\partial(x_n, x_{n+1}, t) + \partial(x_n, x_{n+1}, t)} \right] \end{aligned} \right\} \\ &\varphi \left\{ \begin{aligned} &a \text{Max}[\partial(x_n, x_{n+1}, t), \partial(x_{n+1}, x_{n+2}, t)] + b[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] \\ &\quad + c[\partial(x_n, x_{n+2}, t) + \partial(x_{n+1}, x_{n+1}, t)] \\ &\quad + d \left[ \frac{\partial(x_n, x_{n+1}, t) \cdot \partial(x_{n+1}, x_{n+1}, t)}{\partial(x_n, x_{n+1}, t) + \partial(x_n, x_{n+1}, t)} \right] + e \left[ \frac{\partial(x_n, x_{n+1}, t) \cdot \partial(x_{n+1}, x_{n+2}, t)}{\partial(x_n, x_{n+1}, t) + \partial(x_n, x_{n+1}, t)} \right] \end{aligned} \right\} \end{aligned} \right] \\ &\leq \mathbb{F} \left[ \begin{aligned} &\phi \left\{ \begin{aligned} &a \text{Max}[\partial(x_n, x_{n+1}, t), \partial(x_{n+1}, x_{n+2}, t)] + b[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] \\ &\quad + c\partial(x_n, x_{n+2}, t) + e \left[ \frac{\partial(x_n, x_{n+1}, t) \cdot \partial(x_{n+1}, x_{n+2}, t)}{2\partial(x_n, x_{n+1}, t)} \right] \end{aligned} \right\} \\ &\varphi \left\{ \begin{aligned} &a \text{Max}[\partial(x_n, x_{n+1}, t), \partial(x_{n+1}, x_{n+2}, t)] + b[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] \\ &\quad + c\partial(x_n, x_{n+2}, t) + e \left[ \frac{\partial(x_{n+1}, x_{n+2}, t)}{2} \right] \end{aligned} \right\} \end{aligned} \right] \\ &\leq \phi \left\{ \begin{aligned} &a \text{Max}[\partial(x_n, x_{n+1}, t), \partial(x_{n+1}, x_{n+2}, t)] + b[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] \\ &\quad + c\partial(x_n, x_{n+2}, t) + e \left[ \frac{\partial(x_{n+1}, x_{n+2}, t)}{2} \right] \end{aligned} \right\} \end{aligned}$$

There are two cases may be possible:

**Case (I)** if  $\text{Max}[\partial(x_n, x_{n+1}, t), \partial(x_{n+1}, x_{n+2}, t)] = \partial(x_n, x_{n+1}, t)$  then we get,

$$\phi(\partial(x_{n+1}, x_{n+2}, t)) \leq \phi \left\{ \begin{aligned} &a \partial(x_n, x_{n+1}, t) + b[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] \\ &\quad + c\partial(x_n, x_{n+2}, t) + e \left[ \frac{\partial(x_{n+1}, x_{n+2}, t)}{2} \right] \end{aligned} \right\}$$

$$\phi(\partial(x_{n+1}, x_{n+2}, t)) \leq \phi \left\{ \begin{aligned} &a \partial(x_n, x_{n+1}, t) + b[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] \\ &\quad + c[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] + e \left[ \frac{\partial(x_{n+1}, x_{n+2}, t)}{2} \right] \end{aligned} \right\}$$

$$\partial(x_{n+1}, x_{n+2}, t) \leq \left\{ \begin{array}{l} a \partial(x_n, x_{n+1}, t) + b[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] \\ +c[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] + e \left[ \frac{\partial(x_{n+1}, x_{n+2}, t)}{2} \right] \end{array} \right\}$$

$$\partial(x_{n+1}, x_{n+2}, t) \leq (a + b + c)\partial(x_n, x_{n+1}, t) + (b + c + \frac{e}{2})\partial(x_{n+1}, x_{n+2}, t)$$

$$[1 - (b + c + \frac{e}{2})]\partial(x_{n+1}, x_{n+2}, t) \leq (a + b + c)\partial(x_n, x_{n+1}, t)$$

$$\partial(x_{n+1}, x_{n+2}, t) \leq \frac{(a + b + c)}{[1 - (b + c + \frac{e}{2})]} \partial(x_n, x_{n+1}, t)$$

$$\partial(x_{n+1}, x_{n+2}, t) \leq \gamma \partial(x_n, x_{n+1}, t) \text{ Where } \gamma = \frac{(a+b+c)}{[1-(b+c+\frac{e}{2})]}$$

By mathematical induction, we get

$$\partial(x_{n+1}, x_{n+2}, t) \leq \gamma^{n+1} \partial(x_0, x_1, t)$$

for all  $u, v \in N$  with  $v < u$  then we have ,

$$\partial(x_v, x_u, t) \leq \partial(x_v, x_{v+1}, t) + \partial(x_{v+1}, x_u, t)$$

$$\partial(x_v, x_u, t) \leq \partial(x_v, x_{v+1}, t) + \partial(x_{v+1}, x_{v+2}, t) + \partial(x_{v+2}, x_u, t)$$

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$$\partial(x_v, x_u, t) \leq \partial(x_v, x_{v+1}, t) + \partial(x_{v+1}, x_{v+2}, t) + \dots + \partial(x_{u-1}, x_u, t)$$

$$\partial(x_v, x_u, t) = [\gamma^n + \gamma^{n+1} + \dots + \gamma^{u-1}] \partial(x_0, x_1, t)$$

$$\partial(x_v, x_u, t) = \gamma^n [1 + \gamma + \gamma^2 + \dots + \gamma^{u-v-1}] \partial(x_0, x_1, t)$$

$$\partial(x_v, x_u, t) = \gamma^n \frac{(1-\gamma^{u-v})}{(1-\gamma)} \partial(x_0, x_1, t)$$

Since  $\gamma \in [0,1)$ , taking limit as  $u, v \rightarrow \infty$  then we get  $\partial(x_v, x_u, t) \rightarrow 0$ . Hence the sequence  $\{x_n\}$  is a Cauchy sequence. The completeness of  $(X, \partial) \Rightarrow \{x_n\}$  is convergent. Call the limit  $\lambda \in X$ ,

then  $x_n \rightarrow \lambda$  as  $n \rightarrow \infty$  and  $\mathbb{F}_p$  is Continuous then

$$\mathbb{F}_p \lambda = \mathbb{F}_p(\lim_{n \rightarrow \infty} x_n) = \lim_{n \rightarrow \infty} \mathbb{F}_p x_n = \lim_{n \rightarrow \infty} x_{n+1} = \lambda.$$

Hence  $\mathbb{F}_p$  has a Fixed Point in  $X$ .

**Case (II)** if  $Max[\partial(x_n, x_{n+1}, t), \partial(x_{n+1}, x_{n+2}, t)] = \partial(x_{n+1}, x_{n+2}, t)$  then we get,

$$\phi(\partial(x_{n+1}, x_{n+2}, t)) \leq \phi \left\{ \begin{array}{l} a \partial(x_{n+1}, x_{n+2}, t) + b[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] \\ +c\partial(x_n, x_{n+2}, t) + e \left[ \frac{\partial(x_{n+1}, x_{n+2}, t)}{2} \right] \end{array} \right\}$$

$$\phi(\partial(x_{n+1}, x_{n+2}, t)) \leq \phi \left\{ \begin{aligned} & a \partial(x_{n+1}, x_{n+2}, t) + b[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] \\ & + c[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] + e \left[ \frac{\partial(x_{n+1}, x_{n+2}, t)}{2} \right] \end{aligned} \right\}$$

$$\partial(x_{n+1}, x_{n+2}, t) \leq \left\{ \begin{aligned} & a \partial(x_{n+1}, x_{n+2}, t) + b[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] \\ & + c[\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)] + e \left[ \frac{\partial(x_{n+1}, x_{n+2}, t)}{2} \right] \end{aligned} \right\}$$

$$\partial(x_{n+1}, x_{n+2}, t) \leq (b + c)\partial(x_n, x_{n+1}, t) + (a + b + c + \frac{e}{2})\partial(x_{n+1}, x_{n+2}, t)$$

$$[1 - (a + b + c + \frac{e}{2})]\partial(x_{n+1}, x_{n+2}, t) \leq (b + c)\partial(x_n, x_{n+1}, t)$$

$$\partial(x_{n+1}, x_{n+2}, t) \leq \frac{(b + c)}{[1 - (b + c + \frac{e}{2})]} \partial(x_n, x_{n+1}, t)$$

$$\partial(x_{n+1}, x_{n+2}, t) \leq \delta \partial(x_n, x_{n+1}, t) \text{ Where } \delta = \frac{(b+c)}{[1-(b+c+\frac{e}{2})]}$$

By mathematical induction, we get

$$\partial(x_{n+1}, x_{n+2}, t) \leq \delta^{n+1} \partial(x_0, x_1, t)$$

for all  $u, v \in N$  With  $v < u$  then we have ,

$$\partial(x_v, x_u, t) \leq \partial(x_v, x_{v+1}, t) + \partial(x_{v+1}, x_u, t)$$

$$\partial(x_v, x_u, t) \leq \partial(x_v, x_{v+1}, t) + \partial(x_{v+1}, x_{v+2}, t) + \partial(x_{v+2}, x_u, t)$$

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$$\partial(x_v, x_u, t) \leq \partial(x_v, x_{v+1}, t) + \partial(x_{v+1}, x_{v+2}, t) + \dots + \partial(x_{u-1}, x_u, t)$$

$$\partial(x_v, x_u, t) = [\delta^n + \delta^{n+1} + \dots + \delta^{u-1}] \partial(x_0, x_1, t)$$

$$\partial(x_v, x_u, t) = \delta^n [1 + \delta + \delta^2 + \dots + \delta^{u-v-1}] \partial(x_0, x_1, t)$$

$$\partial(x_v, x_u, t) = \delta^n \frac{(1-\delta^{u-v})}{(1-\delta)} \partial(x_0, x_1, t)$$

Since  $\delta \in [0,1)$ , taking limit as  $u, v \rightarrow \infty$  then we get  $\partial(x_v, x_u, t) \rightarrow 0$ . Hence the sequence  $\{x_n\}$  is a Cauchy sequence. The completeness of  $(X, \partial) \Rightarrow \{x_n\}$  is convergent. Call the limit  $m \in X$ ,

then  $x_n \rightarrow m$  and  $\mathbb{F}_p$  is Continuous then

$$\mathbb{F}_p \ell = \mathbb{F}_p(\lim_{n \rightarrow \infty} x_n) = \lim_{n \rightarrow \infty} \mathbb{F}_p x_n = \lim_{n \rightarrow \infty} x_{n+1} = m.$$

Hence  $\mathbb{F}_p$  has a Fixed Point in  $X$ .

**Uniqueness:** Let us consider  $s$  and  $p$  are two fixed point of  $\mathbb{F}_p$

Such that  $\mathbb{F}_p s = s$  and  $\mathbb{F}_p p = p$  Then by above inequality,

$$\phi(\partial(s, p, t)) = \phi(\partial(\mathbb{F}s, \mathbb{F}p, t))$$

$$\leq \mathbb{F} \left[ \begin{array}{l} \phi \left\{ \begin{array}{l} a \text{Max}\{\partial(s, \mathbb{F}_p s, t), \partial(p, \mathbb{F}_p p, t)\} + b[\partial(s, \mathbb{F}_p s, t) + \partial(p, \mathbb{F}_p p, t)] \\ + c[\partial(s, \mathbb{F}_p p, t) + \partial(p, s, t)] \\ + d \left[ \frac{\partial(s, \mathbb{F}_p s, t) \cdot \partial(\mathbb{F}_p s, p, t)}{\partial(s, \mathbb{F}_p s, t) + \partial(s, p, t)} \right] + e \left[ \frac{\partial(s, \mathbb{F}_p s, t) \cdot \partial(p, \mathbb{F}_p p, t)}{\partial(s, \mathbb{F}_p s, t) + \partial(s, p, t)} \right] \end{array} \right\}, \\ \varphi \left\{ \begin{array}{l} a \text{Max}\{\partial(s, \mathbb{F}_p s, t), \partial(p, \mathbb{F}_p p, t)\} + b[\partial(s, \mathbb{F}_p s, t) + \partial(p, \mathbb{F}_p p, t)] \\ + c[\partial(s, \mathbb{F}_p p, t) + \partial(p, s, t)] \\ + d \left[ \frac{\partial(s, \mathbb{F}_p s, t) \cdot \partial(\mathbb{F}_p s, p, t)}{\partial(s, \mathbb{F}_p s, t) + \partial(s, p, t)} \right] + e \left[ \frac{\partial(s, \mathbb{F}_p s, t) \cdot \partial(p, \mathbb{F}_p p, t)}{\partial(s, \mathbb{F}_p s, t) + \partial(s, p, t)} \right] \end{array} \right\} \end{array} \right]$$

$$\leq \mathbb{F} \left[ \begin{array}{l} \phi \left\{ \begin{array}{l} a \text{Max}\{\partial(s, s, t), \partial(p, p, t)\} + b[\partial(s, s, t) + \partial(p, p, t)] \\ + c[\partial(s, p, t) + \partial(p, s, t)] \\ + d \left[ \frac{\partial(s, s, t) \cdot \partial(s, p, t)}{\partial(s, s, t) + \partial(s, p, t)} \right] + e \left[ \frac{\partial(s, s, t) \cdot \partial(p, p, t)}{\partial(s, s, t) + \partial(s, p, t)} \right] \end{array} \right\}, \\ \varphi \left\{ \begin{array}{l} a \text{Max}\{\partial(s, \mathbb{F}_p s, t), \partial(p, \mathbb{F}_p p, t)\} + b[\partial(s, \mathbb{F}_p s, t) + \partial(p, \mathbb{F}_p p, t)] \\ + c[\partial(s, \mathbb{F}_p p, t) + \partial(p, s, t)] \\ + d \left[ \frac{\partial(s, \mathbb{F}_p s, t) \cdot \partial(\mathbb{F}_p s, p, t)}{\partial(x_n, \mathbb{F}_p s, t) + \partial(s, p, t)} \right] + e \left[ \frac{\partial(s, \mathbb{F}_p s, t) \cdot \partial(p, \mathbb{F}_p p, t)}{\partial(s, \mathbb{F}_p s, t) + \partial(s, p, t)} \right] \end{array} \right\} \end{array} \right]$$

$$\leq \mathbb{F} \left[ \begin{array}{l} \phi\{2c \partial(s, p, t)\}, \\ \varphi\{2c \partial(s, p, t)\} \end{array} \right]$$

$$\phi(\partial(s, p, t)) \leq \phi\{2c \partial(s, p, t)\}$$

$$\partial(s, p, t) \leq 2c \partial(s, p, t)$$

$$(1 - 2c)\partial(s, p, t) \leq 0 \Rightarrow \partial(s, p, t) = 0 \Rightarrow s = p.$$

Hence  $\mathbb{F}_p$  has a unique fixed point in  $X$ .

**Theorem (3.2)** let  $(X, \partial)$  be a complete parametric metric space and  $\mathbb{F}_p: X \rightarrow X$  is a continuous Mapping, satisfying the condition:

$$\phi(\partial(\mathbb{F}_p x, \mathbb{F}_p y, t)) \leq DF(\phi(\Omega(x, y)), \varphi(\Omega(x, y)))$$

for all  $x, y \in X, x \neq y$ , for all  $t > 0$  and  $\mathbb{F} \in \mathbb{C}, \phi \in \Psi, \varphi \in \Phi_u, \mathcal{R} \in [0,1)$  and

$$\Omega(x, y) = \text{Max} \left\{ \begin{array}{l} \partial(x, y, t), \partial(\mathbb{F}_p x, y, t), \partial(x, \mathbb{F}_p y, t), \\ \min \left\{ \frac{\partial(x, \mathbb{F}_p x, t) \partial(y, \mathbb{F}_p x, t)}{\partial(x, y, t)}, \frac{\partial(x, \mathbb{F}_p x, t) \partial(y, \mathbb{F}_p y, t) + \partial(x, \mathbb{F}_p y, t) \partial(y, \mathbb{F}_p x, t)}{\partial(x, y, t) + \partial(y, \mathbb{F}_p y, t)} \right\} \end{array} \right\}$$

Then  $\mathbb{F}_p$  has a unique fixed point in  $X$ .

**Proof:** Choose  $x_0 \in X$  and define a sequence  $\{x_n\}_{n \in \mathbb{N}}$  as follows  $\mathbb{F}_p x_n = x_{n+1}$  for  $n = 1, 2, 3 \dots$  then take  $x = x_n$  and  $y = x_{n+1}$  in above equality then

$$\phi(\partial(x_{n+1}, x_{n+2}, t)) = \phi(\partial(\mathbb{F}_p x_n, \mathbb{F}_p x_{n+1}, t))$$

$$\leq \mathcal{D} \mathbb{F} \left[ \left\{ \phi \left\{ \text{Max} \left\{ \min \left\{ \begin{array}{l} \partial(x_n, x_{n+1}, t), \Gamma(\mathbb{F}_p x_n, x_{n+1}, t), \partial(x_n, \mathbb{F}_p x_{n+1}, t), \\ \frac{\partial(x_n, \mathbb{F}_p x_n, t) \partial(x_{n+1}, \mathbb{F}_p x_n, t)}{\partial(x_n, x_{n+1}, t)}, \\ \frac{\partial(x_n, \mathbb{F}_p x_n, t) \partial(x_{n+1}, \mathbb{F}_p x_{n+1}, t) + \partial(x_n, \mathbb{F}_p x_{n+1}, t) \partial(x_{n+1}, \mathbb{F}_p x_{n+1}, t)}{\partial(x_n, x_{n+1}, t) + \partial(x_n, \mathbb{F}_p x_{n+1}, t)} \end{array} \right\} \right\} \right\} \right\} \right]$$

$$\left[ \left\{ \phi \left\{ \text{Max} \left\{ \min \left\{ \begin{array}{l} \partial(x_n, x_{n+1}, t), \partial(\mathbb{F}_p x_n, x_{n+1}, t), \partial(x_n, \mathbb{F}_p x_{n+1}, t), \\ \frac{\partial(x_n, \mathbb{F}_p x_n, t) \partial(x_{n+1}, \mathbb{F}_p x_n, t)}{\Gamma(x_n, x_{n+1}, t)}, \\ \frac{\partial(x_n, \mathbb{F}_p x_n, t) \partial(x_{n+1}, \mathbb{F}_p x_{n+1}, t) + \partial(x_n, \mathbb{F}_p x_{n+1}, t) \partial(x_{n+1}, \mathbb{F}_p x_{n+1}, t)}{\partial(x_n, x_{n+1}, t) + \partial(x_n, \mathbb{F}_p x_{n+1}, t)} \end{array} \right\} \right\} \right\} \right\} \right]$$

$$\leq \mathcal{D} \mathbb{F} \left[ \left\{ \phi \left\{ \text{Max} \left\{ \min \left\{ \begin{array}{l} \partial(x_n, x_{n+1}, t), \partial(x_{n+1}, x_{n+1}, t), \partial(x_n, x_{n+2}, t), \\ \frac{\partial(x_n, x_{n+1}, t) \partial(x_{n+1}, x_{n+1}, t)}{\partial(x_n, x_{n+1}, t)}, \\ \frac{\partial(x_n, x_{n+1}, t) \partial(x_{n+1}, x_{n+2}, t) + \partial(x_n, x_{n+2}, t) \partial(x_{n+1}, x_{n+2}, t)}{\partial(x_n, x_{n+1}, t) + \partial(x_n, x_{n+2}, t)} \end{array} \right\} \right\} \right\} \right\} \right]$$

$$\left[ \left\{ \phi \left\{ \text{Max} \left\{ \min \left\{ \begin{array}{l} \partial(x_n, x_{n+1}, t), \partial(x_{n+1}, x_{n+1}, t), \partial(x_n, x_{n+2}, t), \\ \frac{\partial(x_n, x_{n+1}, t) \partial(x_{n+1}, x_{n+1}, t)}{\partial(x_n, x_{n+1}, t)}, \\ \frac{\partial(x_n, x_{n+1}, t) \partial(x_{n+1}, x_{n+2}, t) + \partial(x_n, x_{n+2}, t) \partial(x_{n+1}, x_{n+2}, t)}{\partial(x_n, x_{n+1}, t) + \partial(x_n, x_{n+2}, t)} \end{array} \right\} \right\} \right\} \right\} \right]$$

$$\leq \mathcal{D} \mathbb{F} \left[ \phi \left\{ \text{Max} \{ \partial(x_n, x_{n+1}, t), 0, \partial(x_n, x_{n+2}, t), 0, 0 \} \right\}, \right. \\ \left. \phi \left\{ \text{Max} \{ \partial(x_n, x_{n+1}, t), 0, \partial(x_n, x_{n+2}, t), 0, 0 \} \right\} \right]$$

$$\leq \mathcal{D} \mathbb{F} \left[ \phi \left\{ \text{Max} \{ \partial(x_n, x_{n+1}, t), 0, \partial(x_n, x_{n+2}, t), 0, 0 \} \right\} \right]$$

$$\leq \mathcal{D} \phi \left\{ \text{Max} \{ \partial(x_n, x_{n+1}, t), 0, \partial(x_n, x_{n+2}, t), 0, 0 \} \right\}$$

$$\leq \mathcal{D} \text{Max} \{ \partial(x_n, x_{n+1}, t), \partial(x_n, x_{n+2}, t) \}$$

Two cases may be possible:

**Case (I)** if  $\text{Max} \{ \partial(x_n, x_{n+1}, t), \partial(x_{n+1}, x_{n+2}, t) \} = \partial(x_n, x_{n+1}, t)$  then we get,

$$\partial(x_{n+1}, x_{n+2}, t) \leq \mathcal{D} \partial(x_n, x_{n+1}, t)$$

By mathematical induction, we get

$$\partial(x_{n+1}, x_{n+2}, t) \leq \mathcal{D}^{n+1} \partial(x_0, x_1, t)$$

for all  $u, v \in N$  with  $v < u$  then we have ,

$$\partial(x_v, x_u, t) \leq \partial(x_v, x_{v+1}, t) + \partial(x_{v+1}, x_u, t)$$

$$\partial(x_v, x_u, t) \leq \partial(x_v, x_{v+1}, t) + \partial(x_{v+1}, x_{v+2}, t) + \partial(x_{v+2}, x_u, t)$$

.....  
 .....

$$\partial(x_v, x_u, t) \leq \partial(x_v, x_{v+1}, t) + \partial(x_{v+1}, x_{v+2}, t) + \dots + \partial(x_{u-1}, x_u, t)$$

$$\partial(x_v, x_u, t) = [D^n + D^{n+1} + \dots + D^{u-1}] \partial(x_0, x_1, t)$$

$$\partial(x_v, x_u, t) = R^n [1 + D + D^2 + \dots + D^{u-v-1}] \partial(x_0, x_1, t)$$

$$\partial(x_v, x_u, t) = D^n \frac{(1-D^{u-v})}{(1-D)} \partial(x_0, x_1, t)$$

Since,  $\in [0,1)$ , taking limit as  $u, v \rightarrow \infty$  then we get  $\partial(x_v, x_u, t) \rightarrow 0$ . Hence the sequence  $\{x_n\}$  is a Cauchy sequence. The completeness of  $(X, \partial) \Rightarrow \{x_n\}$  is convergent. Call the limit  $\lambda \in X$ ,

then  $x_n \rightarrow \lambda$  as  $n \rightarrow \infty$  and  $\mathbb{F}$  is Continuous then

$$\mathbb{F}_p \lambda = \mathbb{F}_p(\lim_{n \rightarrow \infty} x_n) = \lim_{n \rightarrow \infty} \mathbb{F}_p x_n = \lim_{n \rightarrow \infty} x_{n+1} = \lambda$$

Hence  $\mathbb{F}_p$  has a Fixed Point in  $X$ .

**Case (II)** if  $Max\{\partial(x_n, x_{n+1}, t), \partial(x_{n+1}, x_{n+2}, t)\} = \partial(x_n, x_{n+2}, t)$  then we get,

$$\partial(x_{n+1}, x_{n+2}, t) \leq D \partial(x_n, x_{n+2}, t)$$

$$\partial(x_{n+1}, x_{n+2}, t) \leq D [\partial(x_n, x_{n+1}, t) + \partial(x_{n+1}, x_{n+2}, t)]$$

$$(1 - R)\Gamma(x_{n+1}, x_{n+2}, t) \leq D\partial(x_n, x_{n+1}, t)$$

$$\begin{aligned} \partial(x_{n+1}, x_{n+2}, t) &\leq \frac{D}{(1-D)} \partial(x_n, x_{n+1}, t) \\ &\leq \left(\frac{D}{1-D}\right)^2 \partial(x_{n-1}, x_n, t) \end{aligned}$$

.....  
 .....

$$\leq \left(\frac{D}{1-D}\right)^{n+1} \partial(x_0, x_1, t)$$

We can be easily shown that the sequence  $\{x_n\}$  is a Cauchy sequence by using the above result for all  $u, v \in N$  with  $v < u$ . then by completeness of  $(X, \partial) \Rightarrow \{x_n\}$  is convergent. Call the limit  $\eta \in X$ ,

then  $x_n \rightarrow \lambda$  as  $n \rightarrow \infty$  and  $\mathbb{F}_p$  is Continuous then

$$\mathbb{F}_p \eta = \mathbb{F}_p(\lim_{n \rightarrow \infty} x_n) = \lim_{n \rightarrow \infty} \mathbb{F}_p x_n = \lim_{n \rightarrow \infty} x_{n+1} = \eta$$

Hence  $\mathbb{F}_p$  has a Fixed Point in  $X$ .

**Uniqueness:** Let us consider  $\mathcal{C}_1$  and  $\mathcal{C}_2$  are two fixed point of  $\mathbb{F}_p$

Such that  $\mathbb{F}_p \mathcal{C}_1 = \mathcal{C}_1$  and  $\mathbb{F}_p \mathcal{C}_2 = \mathcal{C}_2$ . Then by above inequality,

$$\phi(\partial(\mathcal{E}_1, \mathcal{E}_2, t)) = \phi(\partial(\mathbb{F}_p \mathcal{E}_1, \mathbb{F}_p \mathcal{E}_2, t))$$

$$\leq \mathcal{D} \mathbb{F} \left[ \left\{ \phi \left\{ \text{Max} \left\{ \min \left\{ \begin{array}{l} \partial(\mathcal{E}_1, \mathcal{E}_2, t), \partial(\mathbb{F}_p \mathcal{E}_1, \mathcal{E}_2, t), \partial(\mathcal{E}_1, \mathbb{F}_p \mathcal{E}_2, t), \\ \frac{\partial(\mathcal{E}_1, \mathbb{F}_p \mathcal{E}_1, t) \partial(\mathcal{E}_2, \mathbb{F}_p \mathcal{E}_1, t)}{\partial(\mathcal{E}_1, \mathcal{E}_2, t)}, \\ \frac{\partial(x_n, \mathbb{F}_p x_n, t) \partial(x_{n+1}, \mathbb{F}_p x_{n+1}, t) + \partial(x_n, \mathbb{F}_p x_{n+1}, t) \partial(x_{n+1}, \mathbb{F}_p x_{n+1}, t)}{\partial(x_n, x_{n+1}, t) + \partial(x_n, \mathbb{F}_p x_{n+1}, t)} \end{array} \right\} \right\} \right\} \right\} \right]$$

$$\left[ \left\{ \phi \left\{ \text{Max} \left\{ \min \left\{ \begin{array}{l} \partial(\mathcal{E}_1, \mathcal{E}_2, t), \partial(\mathbb{F}_p \mathcal{E}_1, \mathcal{E}_2, t), \partial(\mathcal{E}_1, \mathbb{F}_p \mathcal{E}_2, t), \\ \frac{\partial(\mathcal{E}_1, \mathbb{F}_p \mathcal{E}_1, t) \partial(\mathcal{E}_2, \mathbb{F}_p \mathcal{E}_1, t)}{\partial(\mathcal{E}_1, \mathcal{E}_2, t)}, \\ \frac{\partial(\mathcal{E}_1, \mathbb{F}_p \mathcal{E}_1, t) \partial(\mathcal{E}_2, \mathbb{F}_p \mathcal{E}_2, t) + \partial(\mathcal{E}_1, \mathbb{F}_p \mathcal{E}_2, t) \partial(\mathcal{E}_2, \mathbb{F}_p \mathcal{E}_2, t)}{\partial(\mathcal{E}_1, \mathcal{E}_2, t) + \partial(\mathcal{E}_1, \mathbb{F}_p \mathcal{E}_2, t)} \end{array} \right\} \right\} \right\} \right\} \right]$$

$$\leq \mathcal{D} \mathbb{F} \left[ \left\{ \phi \left\{ \text{Max} \left\{ \min \left\{ \begin{array}{l} \partial(\mathcal{E}_1, \mathcal{E}_2, t), \partial(\mathcal{E}_1, \mathcal{E}_2, t), \partial(\mathcal{E}_1, \mathcal{E}_2, t), \\ \frac{\partial(\mathcal{E}_1, \mathcal{E}_1, t) \partial(\mathcal{E}_2, \mathcal{E}_1, t)}{\partial(\mathcal{E}_1, \mathcal{E}_2, t)}, \\ \frac{\partial(\mathcal{E}_1, \mathcal{E}_1, t) \partial(\mathcal{E}_2, \mathcal{E}_2, t) + \partial(\mathcal{E}_1, \mathcal{E}_2, t) \partial(\mathcal{E}_2, \mathcal{E}_2, t)}{\partial(\mathcal{E}_1, \mathcal{E}_2, t) + \partial(\mathcal{E}_1, \mathcal{E}_2, t)} \end{array} \right\} \right\} \right\} \right\} \right]$$

$$\left[ \left\{ \phi \left\{ \text{Max} \left\{ \min \left\{ \begin{array}{l} \partial(\mathcal{E}_1, \mathcal{E}_2, t), \partial(\mathcal{E}_1, \mathcal{E}_2, t), \partial(\mathcal{E}_1, \mathcal{E}_2, t), \\ \frac{\partial(\mathcal{E}_1, \mathcal{E}_1, t) \partial(\mathcal{E}_2, \mathcal{E}_1, t)}{\partial(\mathcal{E}_1, \mathcal{E}_2, t)}, \\ \frac{\partial(\mathcal{E}_1, \mathcal{E}_1, t) \partial(\mathcal{E}_2, \mathcal{E}_2, t) + \partial(\mathcal{E}_1, \mathcal{E}_2, t) \partial(\mathcal{E}_2, \mathcal{E}_2, t)}{\partial(\mathcal{E}_1, \mathcal{E}_2, t) + \partial(\mathcal{E}_1, \mathcal{E}_2, t)} \end{array} \right\} \right\} \right\} \right\} \right]$$

$$\leq \mathcal{D} \mathbb{F} \left[ \phi \left\{ \text{Max} \{ \partial(\mathcal{E}_1, \mathcal{E}_2, t), 0, \partial(\mathcal{E}_1, \mathcal{E}_2, t), 0, 0 \} \right\}, \right. \\ \left. \phi \left\{ \text{Max} \{ \partial(\mathcal{E}_1, \mathcal{E}_2, t), 0, \partial(\mathcal{E}_1, \mathcal{E}_2, t), 0, 0 \} \right\} \right]$$

$$\leq \mathcal{D} \phi \left\{ \text{Max} \{ \{ \partial(\mathcal{E}_1, \mathcal{E}_2, t), 0, \partial(\mathcal{E}_1, \mathcal{E}_2, t), 0, 0 \} \} \right\}$$

$$\leq \mathcal{D} \text{Max} \{ \{ \partial(\mathcal{E}_1, \mathcal{E}_2, t), \partial(\mathcal{E}_1, \mathcal{E}_2, t) \} \}$$

$$\partial(\mathcal{E}_1, \mathcal{E}_2, t) \leq \mathcal{D} \partial(\mathcal{E}_1, \mathcal{E}_2, t)$$

Since,  $\mathcal{D} \in [0, 1)$ , then we get  $\partial(\mathcal{E}_1, \mathcal{E}_2, t) = 0 \Rightarrow \mathcal{E}_1 = \mathcal{E}_2$ .

Hence  $\mathbb{F}_p$  has a unique fixed point in  $X$ .

**Example (3.3)** let  $(X, \partial)$  be a complete parametric metric space and  $\mathbb{F}_p: X \rightarrow X$  is a continuous

Mapping defined as  $\partial(x, y, t) = t|x - y|^p$ ,  $t > 0$ , such that  $x_n = 1 + \frac{1}{n}$  and  $y_n = 1 + \frac{2}{n}$

Then  $\lim_{n \rightarrow \infty} \partial(x_n, y_n, t) = \lim_{n \rightarrow \infty} t|x_n - y_n|^p$

$$= \lim_{n \rightarrow \infty} t \left| \left(1 + \frac{1}{n}\right) - \left(1 + \frac{2}{n}\right) \right|^p$$

$$= \lim_{n \rightarrow \infty} t \left| \frac{1}{n} \right|^p = t \cdot \lim_{n \rightarrow \infty} \frac{1}{n^p} = 0$$

Therefore  $\lim_{n \rightarrow \infty} \partial(x_n, y_n, t) \rightarrow 0$  as both  $x_n = 1 + \frac{1}{n}$  and  $y_n = 1 + \frac{2}{n}$  tends to 1 as  $n \rightarrow \infty$ .

Hence 1 is a fixed point.

**Corollary (3.4)** If we take  $a = 0$  in theorem (3.1) then

let  $(X, \partial)$  be a complete parametric metric space and  $\mathbb{F}_p: X \rightarrow X$  is a continuous

Mapping, satisfying the condition:

$$\phi \left( \partial(\mathbb{F}_p x, \mathbb{F}_p y, t) \right) \leq \mathbb{F} \left( \phi(\Omega(x, y)), \varphi(\Omega(x, y)) \right)$$

for all  $x, y \in X$ , for all  $t > 0$  and  $\mathbb{F} \in \mathbb{C}$ ,  $\phi \in \Psi$ ,  $\varphi \in \Phi_u$ ,

$$\Omega(x, y) = b[\partial(x, \mathbb{F}_p x, t) + \partial(y, \mathbb{F}_p y, t)] + c[\partial(x, \mathbb{F}_p y, t) + \partial(y, \mathbb{F}_p x, t)]$$

$$d \left[ \frac{\partial(x, \mathbb{F}_p x, t) \cdot \partial(\mathbb{F}_p x, y, t)}{\partial(x, \mathbb{F}_p x, t) + \partial(x, y, t)} \right] + e \left[ \frac{\partial(x, \mathbb{F}_p x, t) \cdot \partial(y, \mathbb{F}_p y, t)}{\partial(x, \mathbb{F}_p x, t) + \partial(x, y, t)} \right]$$

Where,  $b + c + e < 1$  and  $a, b, c, d, e \in \left[0, \frac{1}{2}\right)$ . Then  $\mathbb{F}_p$  has a unique fixed point in  $X$ .

**Corollary (3.5)** if we take  $a = e = 0$  in theorem (3.1) then

let  $(X, \partial)$  be a complete parametric metric space and  $\mathbb{F}_p: X \rightarrow X$  is a continuous

Mapping, satisfying the condition:

$$\phi \left( \partial(\mathbb{F}_p x, \mathbb{F}_p y, t) \right) \leq \mathbb{F} \left( \phi(\Omega(x, y)), \varphi(\Omega(x, y)) \right)$$

for all  $x, y \in X$ , for all  $t > 0$  and  $\mathbb{F} \in \mathbb{C}$ ,  $\phi \in \Psi$ ,  $\varphi \in \Phi_u$ ,

$$\Omega(x, y) = b[\partial(x, \mathbb{F}_p x, t) + \partial(y, \mathbb{F}_p y, t)] + c[\partial(x, \mathbb{F}_p y, t) + \partial(y, \mathbb{F}_p x, t)]$$

$$+ d \left[ \frac{\partial(x, \mathbb{F}_p x, t) \cdot \partial(\mathbb{F}_p x, y, t)}{\partial(x, \mathbb{F}_p x, t) + \partial(x, y, t)} \right]$$

Where,  $b + c < 1$  and  $a, b, c, d, e \in \left[0, \frac{1}{2}\right)$ . Then  $\mathbb{F}_p$  has a unique fixed point in  $X$ .

**Corollary (3.6)** If we take  $a = d = e = 0$  in theorem (3.1) then we obtain **Ozgur theorem 5[7]**

let  $(X, \partial)$  be a complete parametric metric space and  $\mathbb{F}_p: X \rightarrow X$  is a continuous

Mapping, satisfying the condition:

$$\phi \left( \partial(\mathbb{F}_p x, \mathbb{F}_p y, t) \right) \leq \mathbb{F} \left( \phi(\Omega(x, y)), \varphi(\Omega(x, y)) \right)$$

for all  $x, y \in X$ , for all  $t > 0$  and  $\mathbb{F} \in \mathbb{C}$ ,  $\phi \in \Psi$ ,  $\varphi \in \Phi_u$ ,

$$\Omega(x, y) = b[\partial(x, \mathbb{F}_p x, t) + \partial(y, \mathbb{F}_p y, t)] + c[\partial(x, \mathbb{F}_p y, t) + \partial(y, \mathbb{F}_p x, t)]$$

Where,  $b + c < 1$  and  $a, b, c, d, e \in \left[0, \frac{1}{2}\right)$ . Then  $\mathbb{F}_p$  has a unique fixed point in  $X$ .

#### Conclusion:

In this research paper, we have improved, extended and generalized the U Singh and Naval Singh theorem for new rational contractive conditions and prove fixed point theorem for Parametric metric spaces with the assistance of C-class function. The result of our proposed work can be further extended for other metric spaces and fuzzy metric space.

#### 4. References:

1. Ansari, A.H., Kumar, J., Vashistha, S., C-class function on Common Fixed point theorem of Weakly Compatible Maps in Partial Metric space. *International Journal of Advances in Mathematics*, 3 (2019) pp-15-23.
2. Ansari, A.H., Note on  $\phi - \phi$ -contractive type mapping and related fixed point theorem. *In the 2<sup>nd</sup> Regional conference on Mathematics. and Application., Payame Noor University, Tehran, Iran*, (2014) pp-377-380.
3. Choudhary, B.S., Maity, P., Coupled fixed point theorem in generalized metric spaces., *Mathematical and Computer Modelling*, 54 (2011) 73-79.
4. Choudhary, R., Garg, A.K., Fixed point results in parametric metric space., *International Journal on Emerging Technologies*, 10(2b) (2019) 100-104.
5. Daheriya, R.D., Jain, R., Ughade, M., Fixed point, coincidence point and common fixed point theorems under various expansive conditions in parametric metric space and parametric b-metric spaces. *Gazi University Journal of Science*, 29 (2016), 95-107.
6. Daheriya, R.D., Shrivastva, S., Ughade, M., Parametric metric space, parametric b-metric space and expansive typing mapping. *International Journal of Mathematics And its Applications*, 4 (2016) 107-117.
7. Ege, O., De la Sen M., A new perspective on Parametric Metric spaces.  *$\Sigma$ -mathematics*, 7 (2019) 1008.
8. Ege, O., karaca, I., Fixed point theorems and an application in parametric metric space., *Azerbaijan Journal of Mathematics*, 7 (2017) 27-39.
9. Garg, A.K., Ahirwar, S., Parametric metric space and fixed point theorems, *International Journal of research in applied, Natural and social sciences*, 10 (2022) 49-56.
10. Hussain, N., Khaleghizadeh, S., Salimi, P., Abdou, A.A.N., A new approach to fixed point results in triangular intuitionistic fuzzy metric spaces. *Abstract and Applied Analysis* (2014).
11. Hussain, N., Salimi, P., Parvaneh, V., Fixed point results for various contractions in parametric and fuzzy b-metric spaces., *J. Nonlinear Sci. Appl*, 8 (2015) 719-739.
12. Krishnakumar, R.; Sanaatammappa, N.P., Some fixed point theorem in parametric metric space., *International Journal of Mathematics Research*, 8 (2016) 213-220.
13. Krishnakumar, R., Sanaatammappa, N.P., Some fixed point theorem in parametric b-metric space., *Int. J. Math. Sci. Eng. Appl*, 10 (2016) 99-106.
14. Likhitkar, M., Daheriya, R.D., Ughade, M., Common fixed point theorems in parametric metric space under nonlinear type contractions. *Int. J. Math. Arch.*, 7 (2016) 105-109.

15. Mohammod, M. et.al., Coupled fixed point theorem and dislocated quasi-metric space, *Int. J., of pure and app. Math.*, 19 (2018) 1249-1260.
16. Posul, H., Kutukcu, S., On parametric spaces. *Bull. Math. and Stat. Res.*, 5 (2017) 17-20.
17. Priyobarta, N., Yumnam, R., Radenović S.N., Fixed point theorems on Parametric A-metric Spaces, *American J. of App. Math. and Stat.*, 6 (2018) 1-5.
18. Rao, K.P.R., Babu, D.V., Ramudu, E.T., Some unique common fixed point theorem in parametric s-metric space, *Int. J. Innov. Res. Sci. Eng. Technol*, 3 (2014).
19. Shatanawi, W., Abbas, M., Nazir, T., Common coupled coincidence and coupled fixed point results in two generalized metric spaces. *Fixed point theory Appl.*, (2011)
20. Shatanawi, W., Coupled fixed point theorem in generalized metric spaces. *Hacet. J. Math. and Stat.*, 40 (2011) 441-447.
21. Tas, N., Özgür N.Y., On parametric S-metric spaces and fixed point type theorems for expansive mapping, *J. Math.* (2016).
22. Tas, N. ,Ozgur, N.Y, some fixed point results on parametric  $N_b$ -metric space, *Commun. korean Math. Soc.*, 33 (2018) 943-960.
23. Singh, U., Singh. N., A New Presumptive on Parametric Metric Space Via C-Class Function, *Sambodhi* 43 (2020).
24. Singh, U. et. al., Fixed Point Theorems on Parametric Metric Space and b-Parametric Metric Spaces, *International Journal of Engineering Research in Current Trends*, 2 (2020).

