# INTEGRAL INVOLVING EXTENDED JACOBI POLYNOMIAL AND THE MULTIVARIABLE H-FUNCTION

**Dr. PAWAN AGRAWAL**
Department of Mathematics, Raj Rishi College,Alwar (Rajasthan)

*Abstract*—**In this paper we establish finite integral which are believed to be new .Our integral involve the product of the extended Jacobi polynomials and the multivariable H-function on account of the general nature of the function and polynomials occurring in these integral our findings provide interesting extensions of a large number of results.**

**I. INTRODUCTION :** TO UNIFY THE CLASSICAL ORTHOGONAL POLYNOMIALS VIZ.JACOBI, HERMITE AND LAGUERRE FUJIWARA[2] DEFINED A CLASS OF GENERALIZED CLASSICAL POLYNOMIALS BY MEANS OF FOLLOWING RODRIGUES FORMULA:

$$\mathrm{Rn}(X)=\frac{(-1)^n\,k^n}{n!(x-p)^{\beta}(q-x)^{\alpha}}\frac{d^n}{dx^n}[(x-p)^{\beta+n}\,(q-x)^{\alpha+n}],\ p<x<q,\alpha>-1,\beta>-1 \tag{1.1}$$

Denote these polynomials by Fn (β, α; x) and call them extended Jacobi polynomials Thakare [6] obtained the following form of Rn(x)=Fn(β, α; x)

$$\mathrm{Fn}(\beta,\alpha;x)=\frac{(-1)^n\,K^n\,(q-x)^n\,(1+\beta)_n}{n!}\,{}_2F_1\left[\begin{matrix}-n,-n-\alpha; & \frac{p-x}{q-x}\\ 1+\beta; & \end{matrix}\right]\ p<x<q \tag{1.2}$$

Fujiwara [2] proved that when p=-1,q=1 and $k=\frac{1}{2}$

$$\mathrm{Fn}\,(\beta,\alpha;x)=P_n^{(\alpha,\beta)}(x)\ \text{Where}\ \ P_n^{(\alpha,\beta)}(x)=\frac{(1+\beta)_n}{n!}\left(\frac{x-1}{2}\right)^n{}_2F_1\left[\begin{matrix}-n,-n-\alpha; & \frac{x+1}{x-1}\\ 1+\beta; & \end{matrix}\right]\text{is Jacobi polynomial [3]} \tag{1.3}$$

The multivariable H- function occurring in the paper will be defined and represented in the following form [4,pp,251-252,eqn.(C.1)-(C.3)]

$$H\begin{bmatrix}z_1\\ \vdots\\ z_r\end{bmatrix}=H^{\,0,n\,:\,m_1,n_1;\,\dots\dots;\,m_r,n_r}_{\,p,q\,:\,p_1,q_1;\,\dots\dots;\,p_r,q_r}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\,\middle|\,\begin{matrix}(a_j;\,\alpha_j^{(1)},\dots,\alpha_j^{(r)})_{1,p};\ (c_j^{(1)},\gamma_j^{(1)})_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\,\beta_j^{(1)},\dots,\beta_j^{(r)})_{1,q};\ (d_j^{(1)},\delta_j^{(1)})_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\dots\int_{L_r}\psi(\varepsilon_1,\dots,\varepsilon_r)\,\phi_1(\varepsilon_1)\dots\phi_r(\varepsilon_r)z_1^{\varepsilon_1}\dots z_r^{\varepsilon_r}d\varepsilon_1\dots d\varepsilon_r \tag{1.4}$$

For the convergence conditions of the integral given by (1.4)and other details of the multivariable H- function we refer to the book by Srivastava etal . [4,pp,252-253,eqns.(C.4)-(C.8)]

**PRELIMINARIES**

In this paper we need the following results :

(i) [1] p.10, eq.(13) Viz $\int_b^a (t-b)^{x-1}(a-t)^{y-1}\,dt=(a-b)^{x+y-1}\,B(x,y),\ \mathrm{Re}(x)>0, Re(y)>0, b<a$ (2.1)

Where B(x,y) is beta function .

(ii) The Hyper Geometric function [3] $_2F_1(a,b;c;z)=\sum_{n=0}^{\infty}\frac{(a)_n(b)_n z^n}{(c)_n\,n!}$ (2.2)

**Main Integral :**

$$\int_p^q (x-p)^t\,(q-x)^{\alpha}\mathrm{Fn}(\beta,\alpha;\,x)H\begin{bmatrix}z_1(x-p)^{u_1}(q-x)^{v_1}\\ \vdots\\ z_r(x-p)^{u_r}(q-x)^{v_r}\end{bmatrix}dx=\frac{(-1)^n K^n(1+\beta)_n(q-p)^{t+\alpha+n+1}}{n!}\sum_{\ell=0}^{\infty}\frac{(-1)^{\ell}(-n-\alpha)_{\ell}}{\ell!(1+\beta)_{\ell}}$$

$$H^{\,0,n+2\,:\,*}_{\,p+2,q+1\,:\,*}\left[\begin{matrix}z_1(q-p)^{u_1+v_1}\\ \vdots\\ z_r(q-p)^{u_r+v_r}\end{matrix}\,\middle|\,\begin{matrix}A & : & *\\ B & : & *\end{matrix}\right] \tag{3.1}$$

Where

$$A=(-t-\ell;u_1\dots,u_r)(-\alpha-n+\ell;v_1\dots,v_r)(a_j;\alpha_j^{(1)},\dots\ \alpha_j^{(r)})_{1,p} \tag{3.2}$$

$$B=\left(b_j;\ \beta_j^{(1)}\dots\beta_j^{(r)}\right)_{1,q}\left(-\alpha-n-t-1;\ u_1+v_1,\dots,u_r+v_r\right) \tag{3.3}$$

Also the asterisk (*) occurring in the right hand side of (3.1) indicates that parameters at these places are the same as the multivariable H-function in (1.4).

The integral (3.1) Is valid under the following conditions :

(i) $(u_i, v_i) \geq 0 \;\; i = 1, \ldots, r$(not all zero simultaneously )

(ii) $\operatorname{Re}(t+\ell)+\sum_{i=1}^{r} \; u_i \min_{i \leq j \leq m_i} \left[\operatorname{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] > -1$

(iii) $\operatorname{Re}(\alpha+n-\ell)+\sum_{i=1}^{r} \; v_i \min_{i \leq j \leq m_i} \left[\operatorname{Re}\left(\frac{d_j^{(i)}}{\delta_j^{(i)}}\right)\right] > -1$

**Proof of (3.1)**

To establish (3.1) replace the multivariable H- function by its Mellin-Barnes contour integral from. Now we interchange the order of x and $\varepsilon_1 \ldots\ldots \varepsilon_r$ integrals (which is permissible under the conditions stated with (3.1) in the result thus obtained and get after a little simplification the left hand side of (3.1)) (say$\Delta$) as:

$$\Delta = \frac{1}{(2\pi\omega)^r} \int_{L1} \ldots\ldots\ldots\ldots \int_{Lr} \emptyset_1(\varepsilon_1) \ldots\ldots \emptyset(\varepsilon_r) \psi(\varepsilon_1 \ldots\ldots \varepsilon_r) z_1^{\varepsilon_1} \ldots\ldots\ldots z_r^{\varepsilon_r}$$

$$\left\{\int_p^q (x-p)^{t+u_1\varepsilon_1+u_r\varepsilon_r}(q-x)^{\alpha+v_1\varepsilon_1+\cdots+v_r\varepsilon_r} Fn(\beta,\alpha;x)dx\right\} d\varepsilon_1 \ldots\ldots d\varepsilon_r \qquad (3.4)$$

Now in the inner integral (3.4) put the value of extended Jacobi polynomial $Fn(\beta,\alpha;x)$ from (1.2)in its series representation with the help of (2.2) and interchange the order of integration and summation (which is permissible under the condition stated with (3.1)). The equation (3.4)takes the following form after a little simplification with the help of known result (2.1)

$$\Delta = \frac{(-1)^n K^n (1+\beta)_n (q-p)^{t+\alpha+n+1}}{n!} \sum_{l=0}^{\infty} \frac{(-1)^l (-n-\alpha)_l}{l!\,(1+\beta)_l}$$

$$\left\{\frac{1}{(2\pi\omega)^r} \int_{L_1} \ldots\ldots \int_{L_r} \emptyset_r(\varepsilon_r) \psi(\varepsilon_1 \ldots\ldots \varepsilon_r) z_1^{\varepsilon_1} \ldots\ldots z_r^{\varepsilon_r} (q-p)^{(u_i+v_i)\varepsilon_i} \frac{\Gamma(t+l+\sum_{i=1}^{r} u_i\varepsilon_i + 1)\Gamma(\alpha+n-l+\sum_{i=1}^{r} v_i\varepsilon_i + 1)}{\Gamma(2+\alpha+n+t+\sum_{i=1}^{r}(u_i+v_i)\varepsilon_i} d\varepsilon_1 \ldots d\varepsilon_r \right\} \qquad (3.5)$$

Finally on reinterpreting the multiple Mellin-Barnes contour integral occurring in right hand side of (3.5)in terms of the multivariable H-function.We arrived at the desired result (3.1).

Special case :If we take p=-1,q=1 and k=$\frac{1}{2}$In (3.1) we get the result which is same as obtained by Saxena and Ramawat [5,p.158,eqn.(2.4)].

**REFERENCES**


1. Erdelyi, A.,1953, ‘‘Higher Transcendental Functions ” Vol. I. McGraw–Hill, New York.
2. Fujiwara, I., 1966, ‘‘A unified presentation of classical orthogonal polynomials,”Math.Japan,11,pp. 133-148.
3. Rainville,E.D., 1960, ‘‘Special functions,” Chelsea Publ.Co. Bronx, New York).
4. Srivastava, H.M.,Gupta, K.C.and Goyal, S.P.,1982, ‘‘The H-function of one and two variables,withapplications,’’ South Asian Publishers,New Delhi.
5. Saxena, R.K. and Ramawat, R.,Jianabha,22(1992),157-164.
6. Thakare, N.K., 1972, ‘‘A study of certain sets of orthogonal polynomials and their applicatons,” Ph.D.Thesis,ShivajiUniv.Kolhapur.